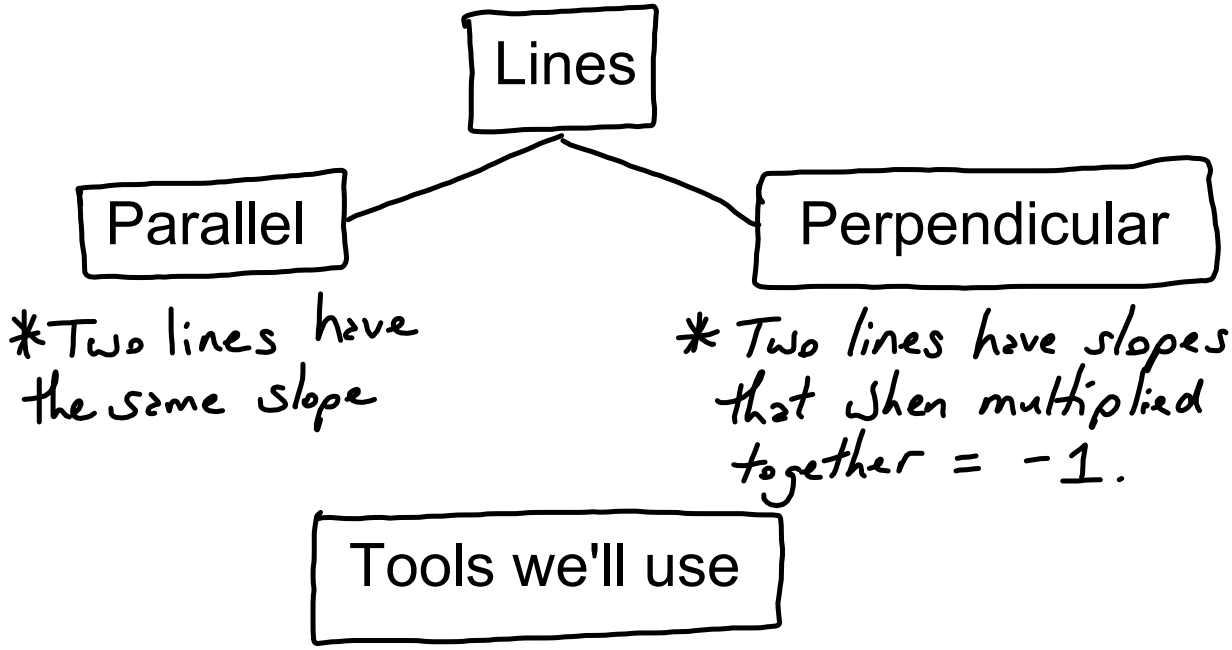
Ch 3.8 - Slopes of Parallel and Perpendicular Lines



Tools we'll use

Yes — Slope $= \frac{y_2 - y_1}{x_2 - x_1}$ — Yes

Yes — Point-Slope Form $y - y_1 = m(x - x_1)$ — Yes

Yes — Slope Intercept Form $y = mx + b$ — Yes

No — Product -1 Form $m_1 \cdot m_2 = -1$ — Yes

3.8 Slopes of Parallel & Perpendicular lines.

Slopes of Parallel lines

* If two nonvertical lines are parallel, then their slopes are equal.
* If the slopes of two nonvertical lines are equal, then the lines are parallel.

* Two vertical lines are always parallel.
* Two horizontal lines are always parallel.

Ex1) Line k has points $(-3,3)$ & $(-1,-4)$ and line ℓ has points $(-1,5)$ & $(2,-4)$. Are line ℓ & line k parallel?

1. Find Slope of each line.

$k = \frac{-4-3}{-1-(-3)} = \frac{-7}{2}$

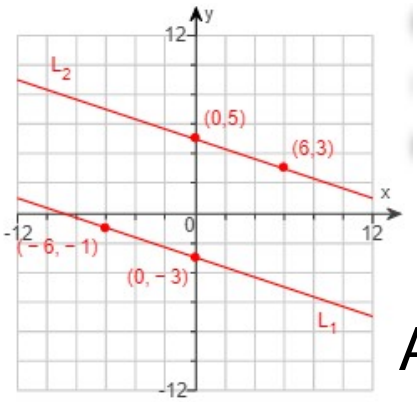
$\ell = \frac{-4-5}{2-(-1)} = \frac{-9}{3}$

2. Compare the Slopes.

Since $\frac{-7}{2} \neq \frac{-9}{3}$, ℓ and k are not parallel.

fractions and signs

$\frac{7}{-2}$ same $\frac{-7}{2}$ same $-\frac{7}{2}$ | $\frac{-7}{-2}$ same $\frac{7}{2}$



Rule: If the slopes of two lines are the same, then the two lines are Parallel.

Are lines L1 and L2 Parallel?

Step 1: Find the slope of each line.

Slope of $L_1 = \frac{-3-(-1)}{0-(-6)}$ $= \frac{-2}{6}$

Slope of $L_2 = \frac{3-5}{6-0}$ $= \frac{-2}{6}$

Step 2: Compare the slopes.

Since $-\frac{2}{6} = \frac{-2}{6}$

lines L_1 + L_2 are parallel

Ex 2)

a) What is an equation of a line parallel to $y = -3x - 5$ and contains the point $(-1, 8)$?

1. Identify the Slope $y = -3x - 5$
2. Use Point-Slope form $y - y_1 = m(x - x_1)$

$y - 8 = -3(x - (-1))$

simplify

$y - 8 = -3(x + 1)$

b) Write the equation in slope-intercept form.

$y - 8 = -3(x + 1)$

$y - 8 = -3x - 3$

$+8 \quad +8$

$y = -3x + 5$

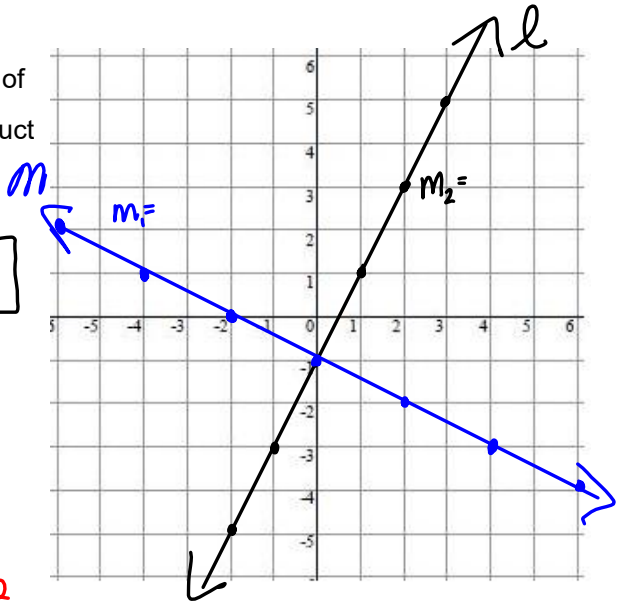
Therefore...

The line $y = -3x - 5$ that passes through point $(-1, 8)$ is parallel to the line $y = -3x + 5$

Slopes of Perpendicular lines

Rule: If the slopes of two lines have a product of -1, then the lines are Perpendicular.

$m_1 \cdot m_2 = -1$



$m_1 = -\frac{1}{2}$

$m_2 = \frac{2}{1}$

$m_1 \cdot m_2$

$-\frac{1}{2} \cdot \frac{2}{1} = -\frac{2}{2}$

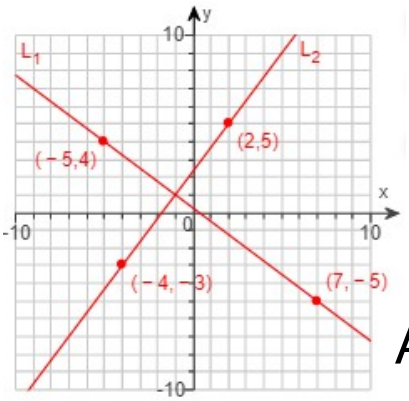
$= -1$

So yes, these two lines are perpendicular

Examples of perpendicular lines Slopes.

$\ell \perp m$

line ℓ slope	line m slope
$\frac{2}{3}$	
$-\frac{1}{8}$	
$-\frac{5}{7}$	
3	



Are lines L1 and L2 perpendicular?

Step 1: Find the slope of each line.

Slope of $L_1 = \frac{-5-4}{7-(-5)}$ Slope of $L_2 = \frac{5-(-3)}{2-(-4)}$

$= \frac{-9}{12}$ $= \frac{8}{6}$

$= -\frac{3}{4}$ $= \frac{4}{3}$

Step 2: Find the product of the slopes.

$m_1 \cdot m_2 = -1$

$-\frac{3}{4} \cdot \frac{4}{3} = -\frac{12}{12}$

$= -1$

Since the product of the two slopes is -1, the lines are Perpendicular.

Ex3| What is an equation of a line perpendicular to $y = \frac{1}{5}x + 2$ and passes through point (15,4)?

1. Identify the slope. $m_1 = \frac{1}{5}$
2. Identify the Perp. slope $m_2 = -5$
3. Use Point-Slope form
$y - y_1 = m(x - x_1)$
$y - 4 = -5(x - 15)$

b) Write the equation in slope-intercept form.

$y - 4 = -5x + 75$
$+4 \qquad +4$
$y = -5x + 79$

Therefore...

Line $y = -5x + 79$ is perpendicular to

Line $y = \frac{1}{5}x + 2$

Ex 4] Write each equation in slope-intercept form, and determine if they are $\parallel$ or $\perp$ or neither.

a) $y+7=-x$ $\qquad$ $y-x=20$

$y=-x-7$ $\qquad$ $y=x+20$

Perpendicular

b) $2x-7y=-42$ $\qquad$ $4y=-7x-2$

$y=\frac{2}{7}x+b$ $\qquad$ $y=\frac{-7}{4}x-\frac{1}{2}$

Neither

c) $6y=4x+24$ $\qquad$ $3y-2x=-3$

$y=\frac{2}{3}x+4$ $\qquad$ $y=\frac{2}{3}x-1$

Parallel