11-3 Surface Areas of Pyramids and Cones

Mathematics Florida Standards

MAFS.912.G-MG.1.1 Use geometric shapes, their measures, and their properties to describe objects.

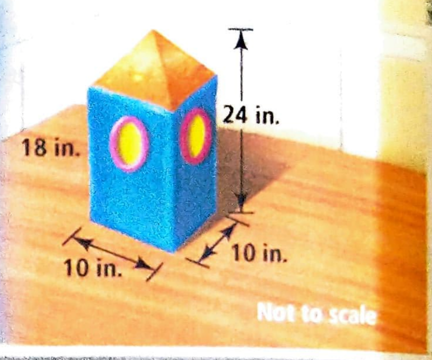
MP 1, MP 3, MP 4, MP 6, MP 7

Objective To find the surface area of a pyramid and a cone

Think about what dimensions you need and how to get them from what you already know.

SOLVE IT! Getting Ready!

You are building a model of a clock tower. You have already constructed the basic structure of the tower at the right. Now you want to paint the roof. How much area does the paint need to cover? Give your answer in square inches. Explain your method.



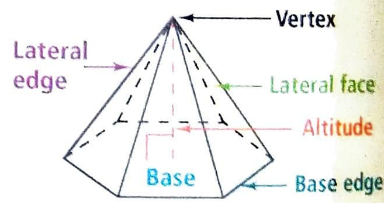
MATHEMATICAL PRACTICES

Lesson Vocabulary
- pyramid (base, lateral face, vertex, altitude, height, slant height, lateral area, surface area)
- regular pyramid
- cone (base, altitude, vertex, height, slant height, lateral area, surface area)
- right cone

The Solve It involves the triangular faces of a roof and the three-dimensional figures they form. In this lesson, you will learn to name such figures and to use formulas to find their areas.

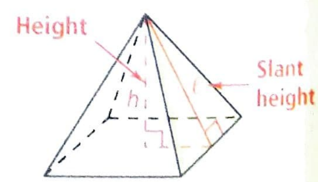
Essential Understanding To find the surface area of a three-dimensional figure, find the sum of the areas of all the surfaces of the figure.

A **pyramid** is a polyhedron in which one face (the **base**) can be any polygon and the other faces (the **lateral faces**) are triangles that meet at a common vertex (called the **vertex** of the pyramid).



Hexagonal pyramid

You name a pyramid by the shape of its base. The **altitude** of a pyramid is the perpendicular segment from the vertex to the plane of the base. The length of the altitude is the **height** h of the pyramid.

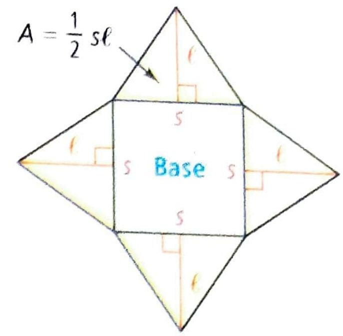


Square pyramid

A **regular pyramid** is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles. The **slant height** ℓ is the length of the altitude of a lateral face of the pyramid.

In this book, you can assume that a pyramid is regular unless stated otherwise.

The **lateral area** of a pyramid is the sum of the areas of the congruent lateral faces. You can find a formula for the lateral area of a pyramid by looking at its net.



$$\text{L.A.} = 4\left(\tfrac{1}{2}s\ell\right)$$ The area of each lateral face is $\frac{1}{2}s\ell$.

$$= \tfrac{1}{2}(4s)\ell$$ Commutative and Associative Properties of Multiplication

$$= \tfrac{1}{2}p\ell$$ The perimeter p of the base is $4s$.

To find the **surface area** of a pyramid, add the area of its base to its lateral area.

take note

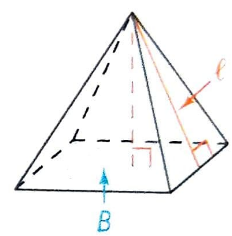
Theorem 11-3 Lateral and Surface Areas of a Pyramid

The lateral area of a regular pyramid is half the product of the perimeter p of the base and the slant height ℓ of the pyramid.

$$\text{L.A.} = \tfrac{1}{2}p\ell$$

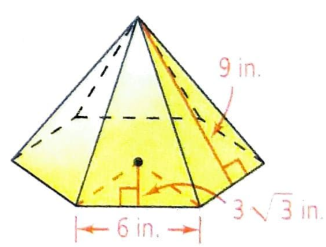
The surface area of a regular pyramid is the sum of the lateral area and the area B of the base.

$$\text{S.A.} = \text{L.A.} + B$$



Problem 1 Finding the Surface Area of a Pyramid

What is the surface area of the hexagonal pyramid?



$\text{S.A.} = \text{L.A.} + B$	Use the formula for surface area.
$= \frac{1}{2}p\ell + \frac{1}{2}ap$	Substitute the formulas for L.A. and B.
$= \frac{1}{2}(36)(9) + \frac{1}{2}\left(3\sqrt{3}\right)(36)$	Substitute.
≈ 255.5307436	Use a calculator.

The surface area of the pyramid is about 256 in.^{2}.

Got It? 1. a. A square pyramid has base edges of 5 m and a slant height of 3 m. What is the surface area of the pyramid?

b. Reasoning Suppose the slant height of a pyramid is doubled. How does this affect the lateral area of the pyramid? Explain.

When the slant height of a pyramid is not given, you must calculate it before you can find the lateral area or surface area.

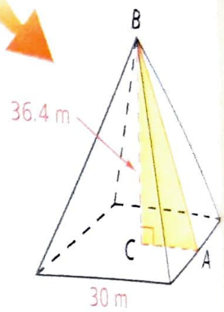
Problem 2 Finding the Lateral Area of a Pyramid

Social Studies The Pyramid of Cestius is located in Rome, Italy. Using the dimensions in the figure below, what is the lateral area of the Pyramid of Cestius? Round to the nearest square meter.

- The height of the pyramid
- The base is a square with a side length of 30 m.
- $\triangle ABC$ is right, where AB is the slant height.

The slant height of the pyramid

Find the perimeter of the base. Use the Pythagorean Theorem to find the slant height. Then use the formula for lateral area.



Step 1 Find the perimeter of the base.

$p = 4s$	Use the formula for the perimeter of a square.
$= 4 \cdot 30$	Substitute 30 for s.
$= 120$	Simplify.

Why is a new diagram helpful for finding the slant height?
The new diagram shows the information you need to use the Pythagorean Theorem.

Step 2 Find the slant height of the pyramid.

BC is the height of the pyramid.

$\overline{CA}$ is the apothem of the base. Its length is $\frac{30}{2}$ m, or 15 m.

The slant height is the length of the hypotenuse of right $\triangle ABC$, or AB.

$\ell = \sqrt{CA^2 + BC^2}$	Use the Pythagorean Theorem.
$= \sqrt{15^2 + 36.4^2}$	Substitute 15 for CA and 36.4 for BC.
$= \sqrt{1549.96}$	Simplify.

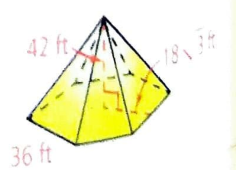
Step 3 Find the lateral area of the pyramid.

$\text{L.A.} = \frac{1}{2}p\ell$	Use the formula for lateral area.
$= \frac{1}{2}(120)\sqrt{1549.96}$	Substitute 120 for p and $\sqrt{1549.96}$ for ℓ.
≈ 2362.171882	Use a calculator.

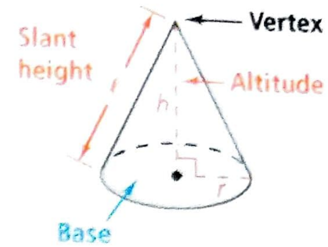
The lateral area of the Pyramid of Cestius is about 2362 m^2.

Got It? 2. a. What is the lateral area of the hexagonal pyramid at the right? Round to the nearest square foot.

b. Reasoning How does the slant height of a regular pyramid relate to its height? Explain.



Like a pyramid, a **cone** is a solid that has one base and a vertex that is not in the same plane as the base. However, the **base** of a cone is a circle. In a **right cone,** the **altitude** is a perpendicular segment from the **vertex** to the center of the base. The **height** h is the length of the altitude. The **slant height** ℓ is the distance from the vertex to a point on the edge of the base. In this book, you can assume that a cone is a right cone unless stated or pictured otherwise.



The **lateral area** is half the circumference of the base times the slant height. The formulas for the lateral area and **surface area** of a cone are similar to those for a pyramid.

take note

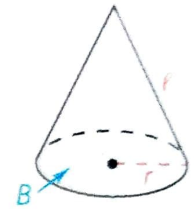
Theorem 11-4 Lateral and Surface Areas of a Cone

The lateral area of a right cone is half the product of the circumference of the base and the slant height of the cone.

$$\text{L.A.} = \frac{1}{2} \cdot 2\pi r \cdot \ell, \text{ or L.A.} = \pi r \ell$$

The surface area of a cone is the sum of the lateral area and the area of the base.

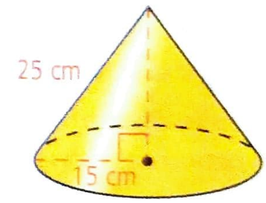
$$\text{S.A.} = \text{L.A.} + B$$



Problem 3 Finding the Surface Area of a Cone

What is the surface area of the cone in terms of π?

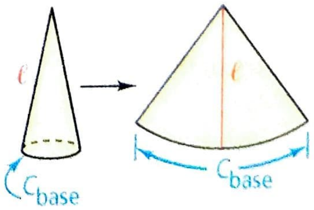
$\text{S.A.} = \text{L.A.} + B$	Use the formula for surface area.
$= \pi r\ell + \pi r^2$	Substitute the formulas for L.A. and B.
$= \pi(15)(25) + \pi(15)^2$	Substitute 15 for r and 25 for ℓ.
$= 375\pi + 225\pi$	Simplify.
$= 600\pi$	



The surface area of the cone is 600π cm^2.

Got It? **3.** The radius of the base of a cone is 16 m. Its slant height is 28 m. What is the surface area in terms of π?

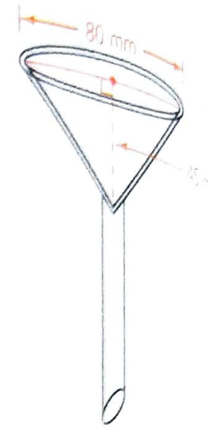
By cutting a cone and laying it out flat, you can see how the formula for the lateral area of a cone $(\text{L.A.} = \frac{1}{2} \cdot C_{base} \cdot \ell)$ resembles that for the area of a triangle $(A = \frac{1}{2}bh)$.



Problem 4 Finding the Lateral Area of a Cone STEM

What is the problem asking you to find?
The problem is asking you to find the area that the filter paper covers. This is the lateral area of a cone.

Chemistry **In a chemistry lab experiment, you use the conical filter funnel shown at the right. How much filter paper do you need to line the funnel?**



The top part of the funnel has the shape of a cone with a diameter of 80 mm and a height of 45 mm.

$\text{L.A.} = \pi r \ell$	Use the formula for lateral area of a cone.
$= \pi r\left(\sqrt{r^2 + h^2}\right)$	To find the slant height, use the Pythagorean Theorem.
$= \pi(40)\left(\sqrt{40^2 + 45^2}\right)$	Substitute $\frac{1}{2} \cdot 80$, or 40, for r and 45 for h.
≈ 7565.957013	Use a calculator.

You need about 7566 mm^2 of filter paper to line the funnel.

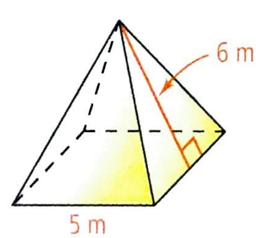
Got It? **4. a.** What is the lateral area of a traffic cone with radius 10 in. and height 28 in.? Round to the nearest whole number.

b. Reasoning Suppose the radius of a cone is halved, but the slant height remains the same. How does this affect the lateral area of the cone? Explain.

Lesson Check

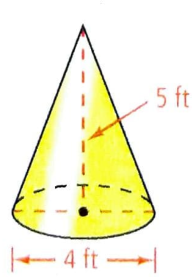
Do you know HOW?

Use the diagram of the square pyramid at the right.



1. What is the lateral area of the pyramid?
2. What is the surface area of the pyramid?

Use the diagram of the cone at the right.



3. What is the lateral area of the cone?
4. What is the surface area of the cone?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** How do the height and the slant height of a pyramid differ?
6. **Compare and Contrast** How are the formulas for the surface area of a prism and the surface area of a pyramid alike? How are they different?
7. **Vocabulary** How many lateral faces does a pyramid have if its base is pentagonal? Hexagonal? n-sided?
8. **Error Analysis** A cone has height 7 and radius 3. Your classmate calculates its lateral area. What is your classmate's error? Explain.

$\text{L.A.} = \pi r \ell$
$= \pi(3)(7)$
$= 21\pi$

Practice and Problem-Solving Exercises

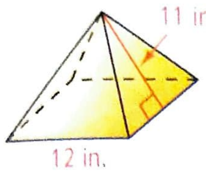
MATHEMATICAL PRACTICES

A Practice

Find the surface area of each pyramid to the nearest whole number.

See Problem 1.

9.



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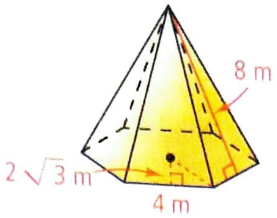
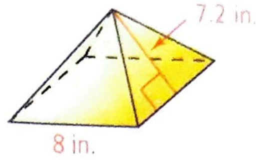


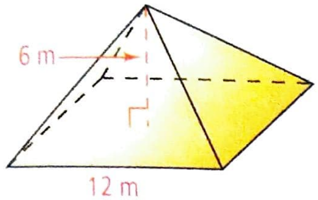
11.



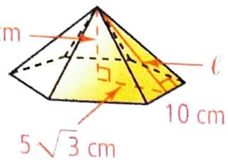
Find the lateral area of each pyramid to the nearest whole number.

See Problem 2.

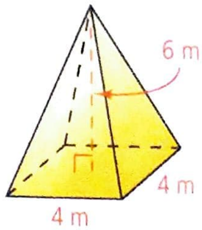
12.



13.



14.

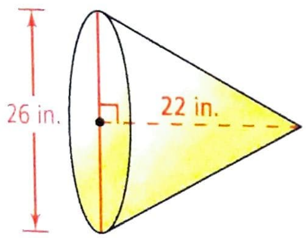


15. Social Studies The original height of the Pyramid of Khafre, located next to the Great Pyramid in Egypt, was about 471 ft. Each side of its square base was about 708 ft. What is the lateral area, to the nearest square foot, of a pyramid with those dimensions?

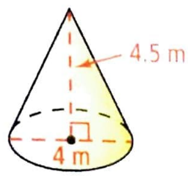
Find the lateral area of each cone to the nearest whole number.

See Problems 3 and 4.

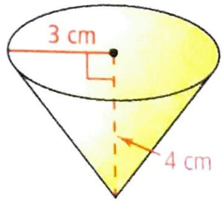
16.



17.



18.

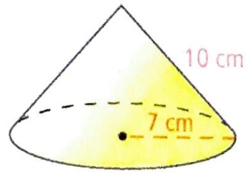


Find the surface area of each cone in terms of π.

19.

20.

21.



B Apply

22. Reasoning Suppose you could climb to the top of the Great Pyramid. Which route would be shorter, a route along a lateral edge or a route along the slant height of a side? Which of these routes is steeper? Explain your answers.

23. The lateral area of a cone is 4.8π in.^{2}. The radius is 1.2 in. Find the slant height.

Practice and Problem-Solving Excercises

9. 408 $in.^2$
10. 138 m^2
11. 179 $in.^2$
12. 204 m^2
13. 354 cm^2
14. 51 m^2
15. 834,308 ft^2
16. 1044 $in.^2$
17. 31 m^2
18. 47 cm^2
19. 144π cm^2
20. 33π ft^2
21. 119π cm^2
22. Slant height; slant height; the slant height is shorter because it is one leg of a rt. △ with the lateral edge as the hypotenuse, and it is steeper because it rises the same vertical distance for a shorter horizontal distance.
23. 4 in.