11-1 Space Figures and Cross Sections

Mathematics Florida Standards
MAFS.912.G-GMD.2.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
MP 1, MP 2, MP 3, MP 4, MP 5, MP 7

Objectives To recognize polyhedrons and their parts
To visualize cross sections of space figures

If you can shift your perspective by reflecting or rotating to get another net, then those nets are the same.

SOLVE IT!

Getting Ready!

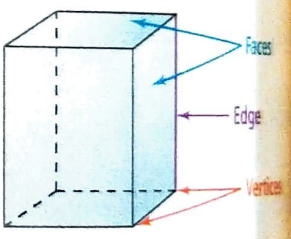
The tissue box at the right is a rectangular solid. Let x = the number of corners, y = the number of flat surfaces, and z = the number of folded creases. What is an equation that relates the quantities x, y, and z for a rectangular solid? Will your equation hold true for a cube? A solid with a triangular top and bottom? Explain.

MATHEMATICAL PRACTICES

In the Solve It, you used two-dimensional nets to represent a three-dimensional object.

Lesson Vocabulary
- polyhedron
- face
- edge
- vertex
- cross section

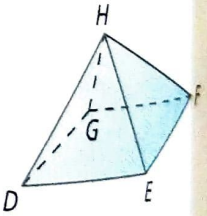
A **polyhedron** is a space figure, or three-dimensional figure, whose surfaces are polygons. Each polygon is a **face** of the polyhedron. An **edge** is a segment that is formed by the intersection of two faces. A **vertex** is a point where three or more edges intersect.



Essential Understanding You can analyze a three-dimensional figure by using the relationships among its vertices, edges, and faces.

Problem 1 Identifying Vertices, Edges, and Faces

How many vertices, edges, and faces are in the polyhedron at the right? List them.



There are five vertices: D, E, F, G, and H.

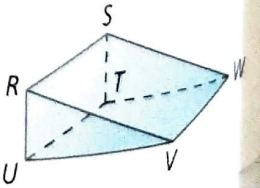
There are eight edges: $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GD}$, $\overline{DH}$, $\overline{EH}$, $\overline{FH}$, and $\overline{GH}$.

There are five faces: $\triangle DEH$, $\triangle EFH$, $\triangle FGH$, $\triangle GDH$, and quadrilateral $DEFG$.

Can you see the solid?
A dashed line indicates an edge that is hidden from view. This figure has one four-sided face and four triangular faces.

Got It? **1. a.** How many vertices, edges, and faces are in the polyhedron at the right? List them.

b. Reasoning Is $\overline{TV}$ an edge? Explain why or why not.



Leonhard Euler, a Swiss mathematician, discovered a relationship among the numbers of faces, vertices, and edges of any polyhedron. The result is known as Euler's Formula.

take note

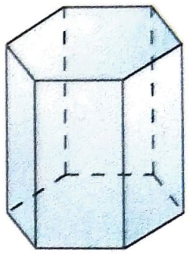
Key Concept Euler's Formula

The sum of the number of faces (F) and vertices (V) of a polyhedron is two more than the number of its edges (E).

$F + V = E + 2$

Problem 2 Using Euler's Formula

How many vertices, edges, and faces does the polyhedron at the right have? Use your results to verify Euler's Formula.



Count the number of faces.

F = 8

Count the number of vertices.

V = 12

Count the number of edges.

E = 18

Substitute the values into Euler's Formula.

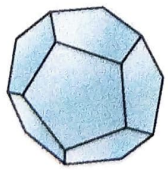
$F + V = E + 2$

$8 + 12 \stackrel{?}{=} 18 + 2$

$20 = 20$ ✓

Got It? 2. For each polyhedron, use Euler's Formula to find the missing number.

a.

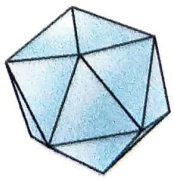


faces: ▪

edges: 30

vertices: 20

b.



faces: 20

edges: ▪

vertices: 12

In two dimensions, Euler's Formula reduces to $F + V = E + 1$, where F is the number of regions formed by V vertices linked by E segments.

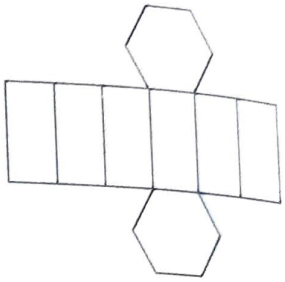
Problem 3 Verifying Euler's Formula in Two Dimensions

What do you use for the variables?

In 3-D		In 2-D
F: Faces	→	Regions
V: Vertices	→	Vertices
E: Edges	→	Segments

How can you verify Euler's Formula for a net for the solid in Problem 2?

Draw a net for the solid.



Number of regions: $F = 8$

Number of vertices: $V = 22$

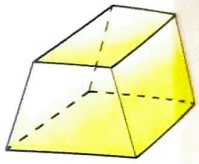
Number of segments: $E = 29$

$F + V = E + 1$ Euler's Formula for two dimensions

$8 + 22 = 29 + 1$ Substitute.

$30 = 30$ ✔

Got It? **3.** Use the solid at the right.

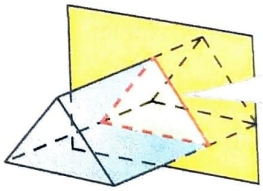


a. How can you verify Euler's Formula $F + V = E + 2$ for the solid?

b. Draw a net for the solid.

c. How can you verify Euler's Formula $F + V = E + 1$ for your two-dimensional net?

A **cross section** is the intersection of a solid and a plane. You can think of a cross section as a very thin slice of the solid.

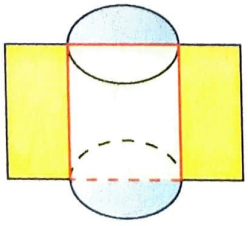


This cross section is a triangle.

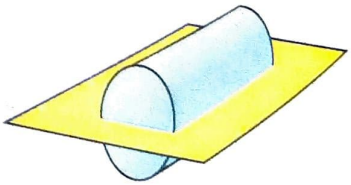
Problem 4 Describing a Cross Section

How can you see the cross section?
Mentally rotate the solid so that the plane is parallel to your face.

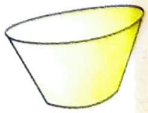
What is the cross section formed by the plane and the solid at the right?



The cross section is a rectangle.



Got It? **4.** For the solid at the right, what is the cross section formed by each of the following planes?

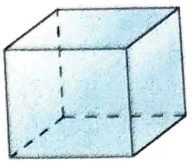


a. a horizontal plane

b. a vertical plane that divides the solid in half

To draw a cross section, you can sometimes use the idea from Postulate 1-3 that the intersection of two planes is exactly one line.

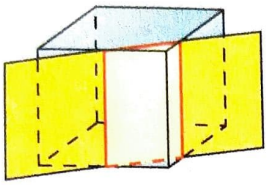
Problem 5 Drawing a Cross Section



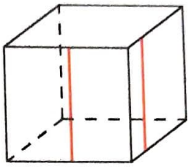
Visualization **Draw a cross section formed by a vertical plane intersecting the front and right faces of the cube. What shape is the cross section?**

Think
How can you see parallel segments?
Focus on the plane intersecting the front and right faces. The plane and both faces are vertical, so the intersections are vertical parallel lines.

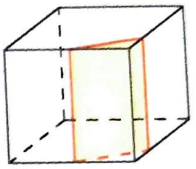
Step 1
Visualize a vertical plane intersecting the vertical faces in parallel segments.



Step 2
Draw the parallel segments.



Step 3
Join their endpoints. Shade the cross section.



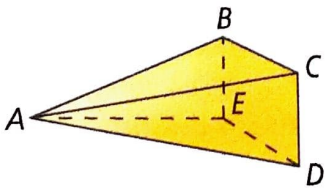
The cross section is a rectangle.

Got It? **5.** Draw the cross section formed by a horizontal plane intersecting the left and right faces of the cube. What shape is the cross section?

Lesson Check

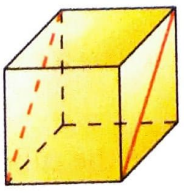
Do you know HOW?

1. How many faces, edges, and vertices are in the solid? List them.



2. What is a net for the solid in Exercise 1? Verify Euler's Formula for the net.

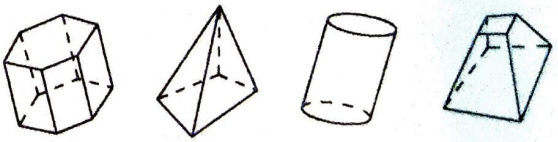
3. What is the cross section formed by the cube and the plane containing the diagonals of a pair of opposite faces?



Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** Suppose you build a polyhedron from two octagons and eight squares. Without using Euler's Formula, how many edges does the solid have? Explain.

5. **Error Analysis** Your math class is drawing polyhedrons. Which figure does not belong in the diagram below? Explain.



Practice and Problem-Solving Exercises

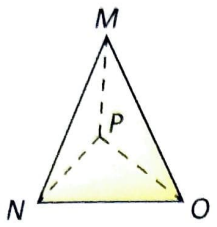
MATHEMATICAL PRACTICES

A Practice

For each polyhedron, how many vertices, edges, and faces are there? List them.

See Proble

6.



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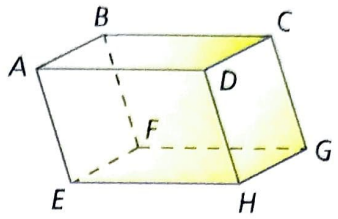

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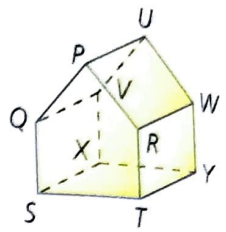


For each polyhedron, use Euler's Formula to find the missing number.

See Probl

9. faces:
edges: 15
vertices: 9

10. faces: 8
edges:
vertices: 6

11. faces: 20
edges: 30
vertices:

Use Euler's Formula to find the number of vertices in each polyhedron.

12. 6 square faces

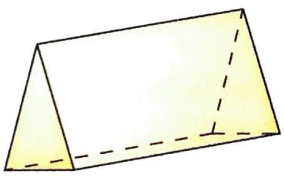
13. 5 faces: 1 rectangle and 4 triangles

14. 9 faces: 1 octagon and 8 triangles

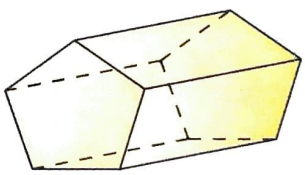
Verify Euler's Formula for each polyhedron. Then draw a net for the figure and verify Euler's Formula for the two-dimensional figure.

See Prob

15.



16.



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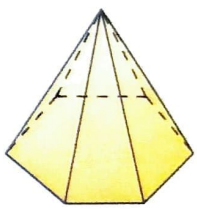
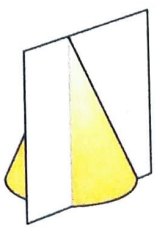


Describe each cross section.

See Pro

18.

19.



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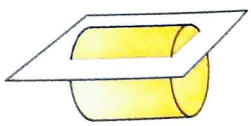

Visualization **Draw and describe a cross section formed by a vertical plane intersecting the cube as follows.**

See Pr

21. The vertical plane intersects the front and left faces of the cube.

22. The vertical plane intersects opposite faces of the cube.

23. The vertical plane contains the red edges of the cube.