11-2 Surface Areas of Prisms and Cylinders

Mathematics Florida Standards

MAFS.912.G-MG.1.1 Use geometric shapes, their measures, and their properties to describe objects.

MP 1, MP 3, MP 4, MP 6, MP 7, MP 8

Objective To find the surface area of a prism and a cylinder

You'll flatten this problem out in no time!

MATHEMATICAL PRACTICES

Getting Ready!

A piece of string is wrapped once around an empty paper towel tube. The ends of the string are attached to each end of the tube as shown. How long is the piece of string? Justify your reasoning.

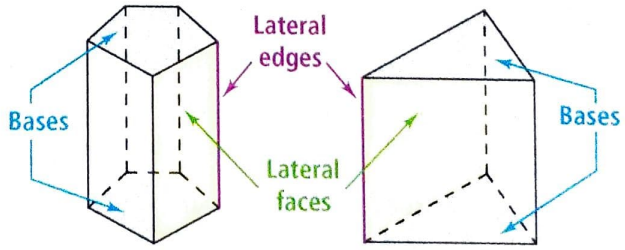
In the Solve It, you investigated the structure of a tube. In this lesson, you will learn properties of three-dimensional figures by investigating their surfaces.

Lesson Vocabulary

- prism (base, lateral face, altitude, height, lateral area, surface area)
- right prism
- oblique prism
- cylinder (base, altitude, height, lateral area, surface area)
- right cylinder
- oblique cylinder

Essential Understanding To find the surface area of a three-dimensional figure, find the sum of the areas of all the surfaces of the figure.

A **prism** is a polyhedron with two congruent, parallel faces, called **bases.** The other faces are **lateral faces.** You can name a prism using the shape of its bases.



Pentagonal prism Triangular prism

An **altitude** of a prism is a perpendicular segment that joins the planes of the bases. The **height** h of a prism is the length of an altitude. A prism may either be right or oblique.

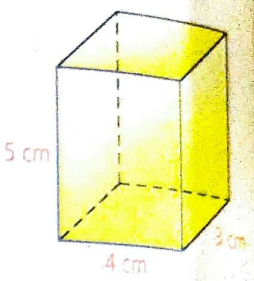
Right prisms Oblique prisms

In a **right prism,** the lateral faces are rectangles and a lateral edge is an altitude. In an **oblique prism,** some or all of the lateral faces are nonrectangular. In this book, you may assume that a prism is a right prism unless stated or pictured otherwise.

The **lateral area** (L.A.) of a prism is the sum of the areas of the lateral faces. The **surface area** (S.A.) is the sum of the lateral area and the area of the two bases.

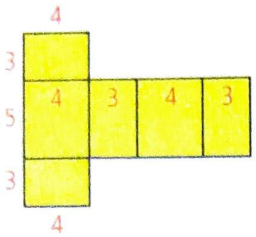
Problem 1 Using a Net to Find Surface Area of a Prism

What is the surface area of the prism at the right? Use a net.



How do you know what units to use?
In the prism, the rectangle marked 5 on one side and 4 on the other has area 5 cm × 4 cm, or 20 cm^2. So use cm^2 as the unit for the surface area of the prism.

Draw a net for the prism. Then calculate the surface area.



S.A. = sum of areas of all the faces

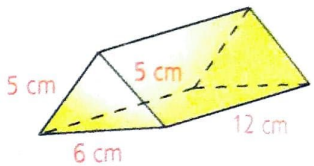
$= 5 \cdot 4 + 5 \cdot 3 + 5 \cdot 4 + 5 \cdot 3 + 3 \cdot 4 + 3 \cdot 4$

$= 20 + 15 + 20 + 15 + 12 + 12$

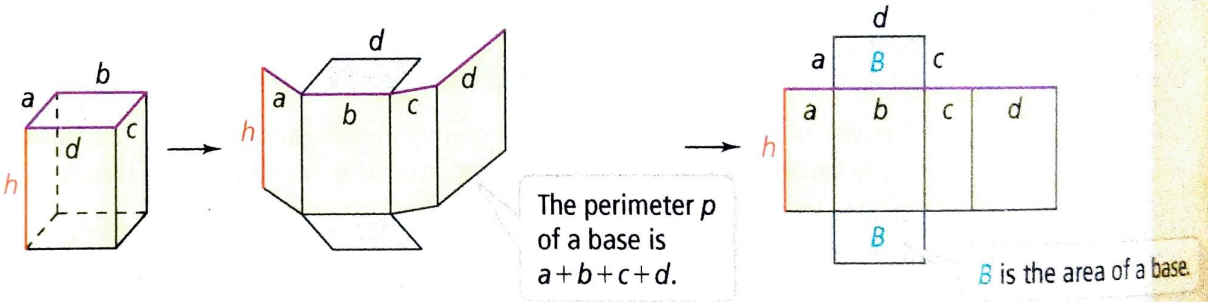
$= 94$

The surface area of the prism is 94 cm^2.

Got It? 1. What is the surface area of the triangular prism? Use a net.



You can find formulas for lateral and surface areas of a prism by using a net.



Lateral Area = ph

Surface Area = L.A. + $2B$

You can use the formulas with any right prism.

take note

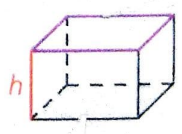
Theorem 11-1 Lateral and Surface Areas of a Prism

The lateral area of a right prism is the product of the perimeter of the base and the height of the prism.

L.A. = ph

The surface area of a right prism is the sum of the lateral area and the areas of the two bases.

S.A. = L.A. + $2B$

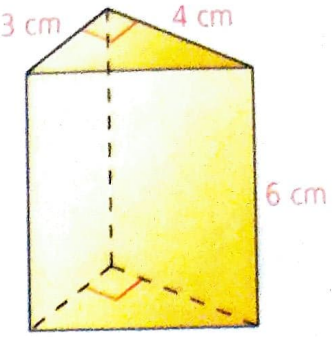


p is the perimeter of a base.

B is the area of a base.

Problem 2 Using Formulas to Find Surface Area of a Prism

What is the surface area of the prism at the right?



Step 1 Find the perimeter of a base.

The perimeter of the base is the sum of the side lengths of the triangle. Since the base is a right triangle, the hypotenuse is $\sqrt{3^2 + 4^2}$ cm, or 5 cm, by the Pythagorean Theorem.

$p = 3 + 4 + 5 = 12$

Step 2 Find the lateral area of the prism.

$\text{L.A.} = ph$ Use the formula for lateral area.

$= 12 \cdot 6$ Substitute 12 for p and 6 for h.

$= 72$ Simplify.

Step 3 Find the area of a base.

$B = \frac{1}{2}bh$ Use the formula for the area of a triangle.

$= \frac{1}{2}(3 \cdot 4)$ Substitute 3 for b and 4 for h.

$= 6$

Step 4 Find the surface area of the prism.

$\text{S.A.} = \text{L.A.} + 2B$ Use the formula for surface area.

$= 72 + 2(6)$ Substitute 72 for L.A. and 6 for B.

$= 84$ Simplify.

The surface area of the prism is 84 cm^2.

What do you need to find first?
You first need to find the missing side length of a triangular base so that you can find the perimeter of a base.

Which height do you need?
For problems involving solids, make it a habit to note which height the formula requires. In Step 3, you need the height of the triangle, not the height of the prism.

Got It? **2. a.** What is the lateral area of the prism at the right?

b. What is the area of a base in simplest radical form?

c. What is the surface area of the prism rounded to a whole number?