11-4 Volumes of Prisms and Cylinders

Mathematics Florida Standards

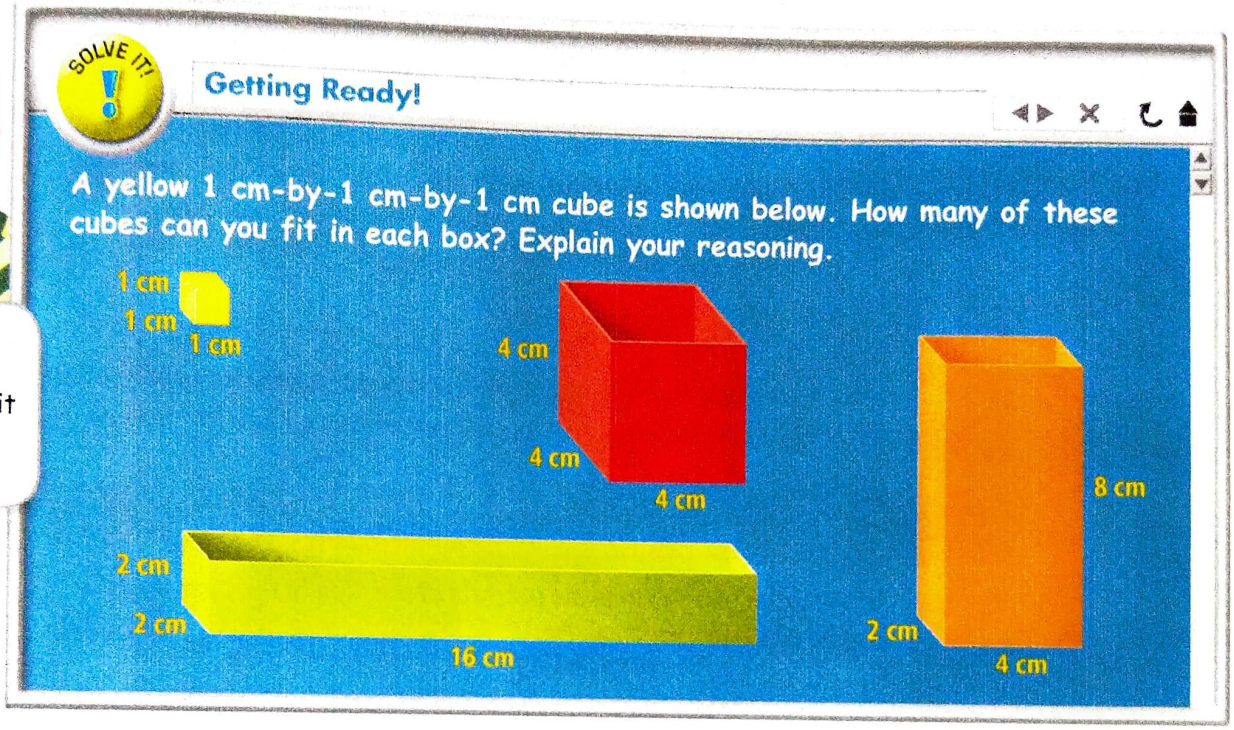
MAFS.912.G-GMD.1.1 Give an informal argument for the formulas for . . . volume of a cylinder . . . Use . . . Cavalieri's principle . . . **Also MAFS.912.G-GMD.1.2, MAFS.912.G-GMD.1.3, MAFS.912.G-MG.1.1**

MP 1, MP 3, MP 4, MP 6, MP 7

Objective To find the volume of a prism and the volume of a cylinder

You can start by figuring out how many cubes will fit on the bottom of the box.

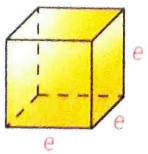
MATHEMATICAL PRACTICES



In the Solve It, you determined the volume of a box by finding how many 1 cm-by-1 cm-by-1 cm cubes the box holds.

Lesson Vocabulary
volume
composite space figure

Volume is the space that a figure occupies. It is measured in cubic units such as cubic inches ($in.^3$), cubic feet (ft^3), or cubic centimeters (cm^3). The volume V of a cube is the cube of the length of its edge e, or $V = e^3$.



Essential Understanding You can find the volume of a prism or a cylinder when you know its height and the area of its base.

Both stacks of paper below contain the same number of sheets.

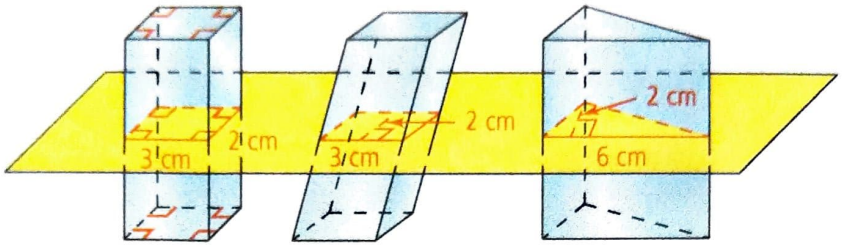
The first stack forms an oblique prism. The second forms a right prism. The stacks have the same height. The area of every cross section parallel to a base is the area of one sheet of paper. The stacks have the same volume. These stacks illustrate the following principle.

take note

Theorem 11-5 Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.

The area of each shaded cross section below is 6 cm^2. Since the prisms have the same height, their volumes must be the same by Cavalieri's Principle.



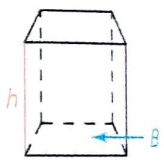
You can find the volume of a right prism by multiplying the area of the base by the height. Cavalieri's Principle lets you extend this idea to any prism.

take note

Theorem 11-6 Volume of a Prism

The volume of a prism is the product of the area of the base and the height of the prism.

$$V = Bh$$

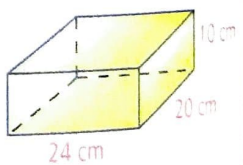


Problem 1 Finding the Volume of a Rectangular Prism

What is the volume of the rectangular prism at the right?

What do you need to use the formula?
You need to find B, the area of the base. The prism has a rectangular base, so the area of the base is length × width.

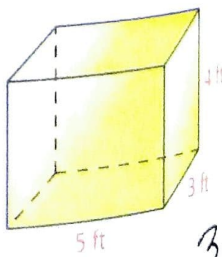
$V = Bh$	Use the formula for the volume of a prism.
$= 480 \cdot 10$	The area of the base B is $24 \cdot 20$, or 480 cm^2, and the height is 10 cm.
$= 4800$	Simplify.



The volume of the rectangular prism is 4800 cm^3.

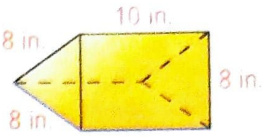
Got It? **1. a.** What is the volume of the rectangular prism at the right?

b. Reasoning Suppose the prism at the right is turned so that the base is 4 ft by 5 ft and the height is 3 ft. Does the volume change? Explain.



Problem 2 Finding the Volume of a Triangular Prism

Multiple Choice What is the approximate volume of the triangular prism?

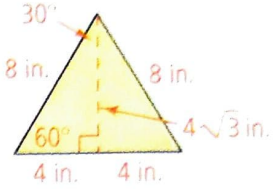


(A) 188 in.^{3} (C) 295 in.^{3}

(B) 277 in.^{3} (D) 554 in.^{3}

Step 1 Find the area of the base of the prism.

Each base of the triangular prism is an equilateral triangle, as shown at the right. An altitude of the triangle divides it into two 30°-60°-90° triangles. The height of the triangle is $\sqrt{3}$ • shorter leg, or $4\sqrt{3}$.



$B = \frac{1}{2}bh$ Use the formula for the area of a triangle.

$= \frac{1}{2}(8)(4\sqrt{3})$ Substitute 8 for b and $4\sqrt{3}$ for h.

$= 16\sqrt{3}$ Simplify.

Step 2 Find the volume of the prism.

$V = Bh$ Use the formula for the volume of a prism.

$= 16\sqrt{3} \cdot 10$ Substitute $16\sqrt{3}$ for B and 10 for h.

$= 160\sqrt{3}$ Simplify.

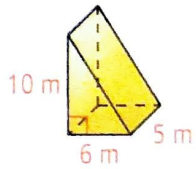
≈ 277.1281292 Use a calculator.

The volume of the triangular prism is about 277 in.^{3}. The correct answer is B.

Which height do you use in the formula? Remember that the h in the formula for volume represents the height of the entire prism, not the height of the triangular base.

Got It? 2. a. What is the volume of the triangular prism at the right?

b. Reasoning Suppose the height of a prism is doubled. How does this affect the volume of the prism? Explain.



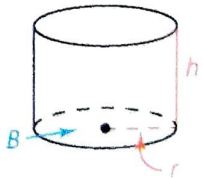
To find the volume of a cylinder, you use the same formula $V = Bh$ that you use to find the volume of a prism. Now, however, B is the area of the circle, so you use the formula $B = \pi r^2$ to find its value.

take note

Theorem 11-7 Volume of a Cylinder

The volume of a cylinder is the product of the area of the base and the height of the cylinder.

$$V = Bh, \text{ or } V = \pi r^2 h$$



Problem 3 Finding the Volume of a Cylinder

What is the volume of the cylinder in terms of π?

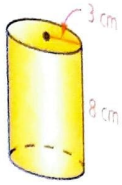
What do you know from the diagram?
You know that the radius r is 3 cm and the height h is 8 cm.

$V = \pi r^2 h$ Use the formula for the volume of a cylinder.

$= \pi(3)^2(8)$ Substitute 3 for r and 8 for h.

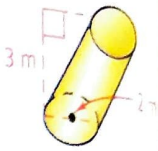
$= \pi(72)$ Simplify.

The volume of the cylinder is 72π cm^3.



Got It? **3. a.** What is the volume of the cylinder at the right in terms of π?

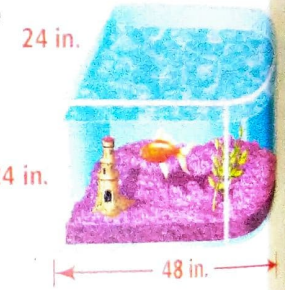
b. Reasoning Suppose the radius of a cylinder is halved. How does this affect the volume of the cylinder? Explain.



A **composite space figure** is a three-dimensional figure that is the combination of two or more simpler figures. You can find the volume of a composite space figure by adding the volumes of the figures that are combined.

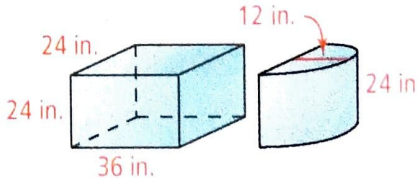
Problem 4 Finding the Volume of a Composite Figure

What is the approximate volume of the bullnose aquarium to the nearest cubic inch?



How can you find the volume by *solving a simpler problem*?
The aquarium is the combination of a rectangular prism and half of a cylinder. Find the volume of each figure.

The length of the prism is the total length minus the radius of the cylinder. The radius of the cylinder is half the width of the prism.



Find the volume of the prism and the half cylinder.

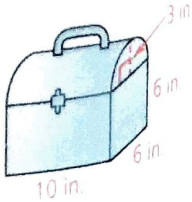
$V_1 = Bh$
$= (24 \cdot 36)(24)$
$= 20{,}736$

$V_2 = \frac{1}{2}\pi r^2 h$
$= \frac{1}{2}\pi(12)^2(24)$
≈ 5429

Add the two volumes together.

$20{,}736 + 5429 = 26{,}165$
The approximate volume of the aquarium is $26{,}165$ in.^{3}.

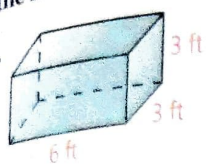
Got It? **4.** What is the approximate volume of the lunchbox shown at the right? Round to the nearest cubic inch.

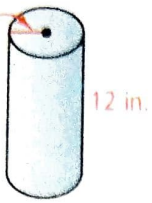


Lesson Check

Do you know HOW?

What is the volume of each figure? If necessary, round to the nearest whole number.

1. 

2. 

Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. **Vocabulary** Is the figure at the right a composite space figure? Explain.

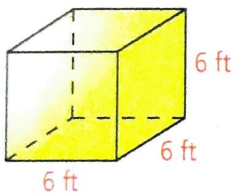
4. **Compare and Contrast** How are the formulas for the volume of a prism and the volume of a cylinder alike? How are they different?

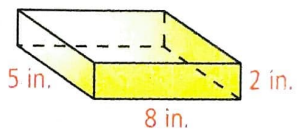
5. **Reasoning** How is the volume of a rectangular prism with base 2 m by 3 m and height 4 m related to the volume of a rectangular prism with base 3 m by 4 m and height 2 m? Explain.

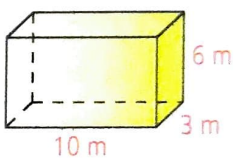
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

Practice

Find the volume of each rectangular prism. See Problem 1.

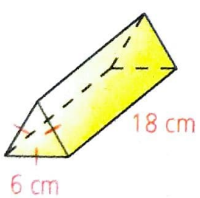
6. 

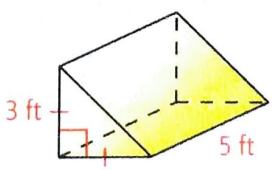
7. 

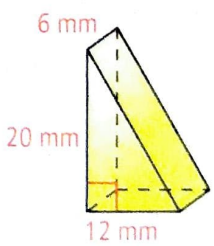
8. 

9. The base is a square with sides of 2 cm. The height is 3.5 cm.

Find the volume of each triangular prism. See Problem 2.

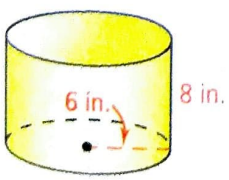
10. 

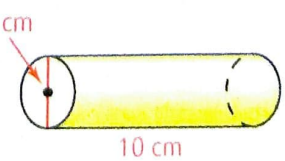
11. 

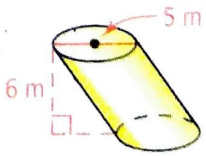
12. 

13. The base is a 45°-45°-90° triangle with a leg of 5 in. The height is 1.8 in.

Find the volume of each cylinder in terms of π and to the nearest tenth. See Problem 3.

14. 

15. 

16. 

17. The diameter of the cylinder is 1 yd. The height is 4 yd.

cylinder

of the
fferent:
ygon,
is a

on is

Practice and Problem-Solving Exercises

6. 216 ft^3

7. 80 $in.^3$

8. 180 m^3

9. 14 cm^3

10. about 280.6 cm^3

11. 22.5 ft^3

12. 720 mm^3

13. 22.5 $in.^3$

14. 288π $in.^3$; 904.8 $in.^3$

15. 40π cm^3; 125.7 cm^3

16. 37.5π m^3; 117.8 m^3

17. π yd^3; 3.1 yd^3