

11-5

Volumes of Pyramids and Cones

Mathematics Florida Standards
MAFS.912.G-GMD.1.3 Use volume formulas for pyramids, cones . . . to solve problems.
MAFS.912.G-MG.1.1 Use geometric shapes, their measures, and their properties to describe objects.
MP 1, MP 3, MP 4, MP 7

Objective To find the volume of a pyramid and of a cone

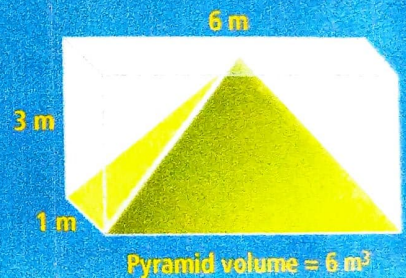
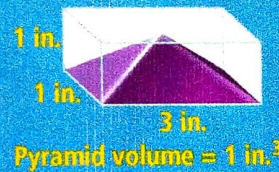
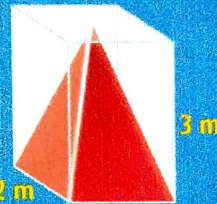


Make a table and look for a pattern.



Getting Ready!

Look for a pattern in the volumes of the prism and pyramid pairs below. Use the pattern to find the volume of a pyramid with a base 2 ft by 3 ft and height 5 ft. Explain your reasoning.



Not to scale

In the Solve It, you analyzed the relationship between the volume of a prism and the volume of an embedded pyramid.

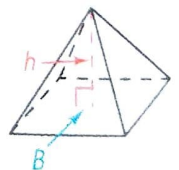
Essential Understanding The volume of a pyramid is related to the volume of a prism with the same base and height.

Take note

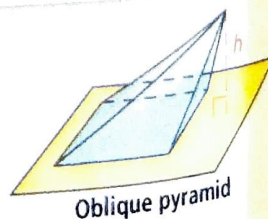
Theorem 11-8 Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

$$V = \frac{1}{3}Bh$$



Because of Cavalieri's Principle, the volume formula is true for all pyramids. The height h of an oblique pyramid is the length of the perpendicular segment from its vertex to the plane of the base.



Problem 1 Finding Volume of a Pyramid **STEM**

Architecture The entrance to the Louvre Museum in Paris, France, is a square pyramid with a height of 21.64 m. What is the approximate volume of the Louvre Pyramid?

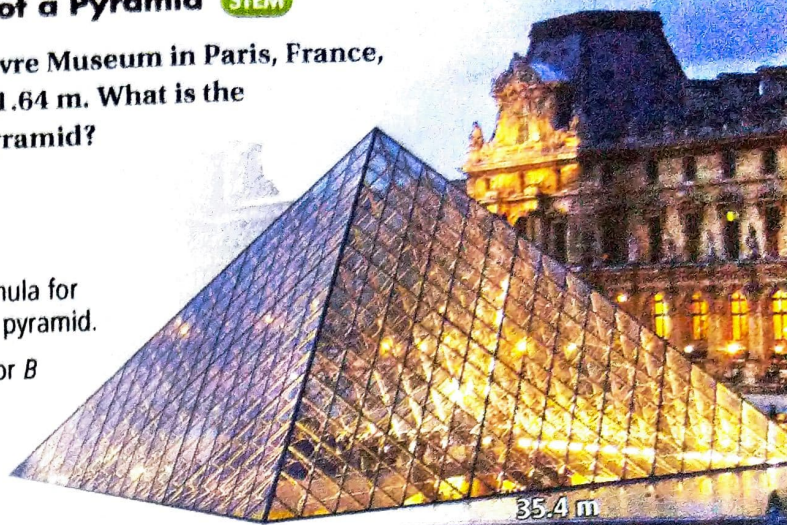
The area of the base of the pyramid is $35.4 \text{ m} \cdot 35.4 \text{ m}$, or 1253.16 m^2 .

$V = \frac{1}{3}Bh$ Use the formula for volume of a pyramid.

$= \frac{1}{3}(1253.16)(21.64)$ Substitute for B and h .

$= 9039.4608$ Simplify.

The volume is about 9039 m^3 .



Got It? 1. A sports arena shaped like a pyramid has a base area of about $300,000 \text{ ft}^2$ and a height of 321 ft. What is the approximate volume of the arena?

Problem 2 Finding the Volume of a Pyramid **GRIDDED RESPONSE**

What is the volume in cubic feet of a square pyramid with base edges 40 ft and slant height 25 ft?

Step 1 Find the height of the pyramid.

$c^2 = a^2 + b^2$ Use the Pythagorean Theorem.

$25^2 = h^2 + 20^2$ Substitute 25 for c , h for a , and $\frac{40}{2}$, or 20, for b .

$625 = h^2 + 400$ Simplify.

$h^2 = 225$ Solve for h^2 .

$h = 15$ Take the positive square root of both sides.

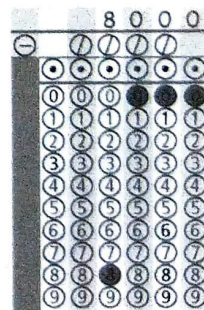
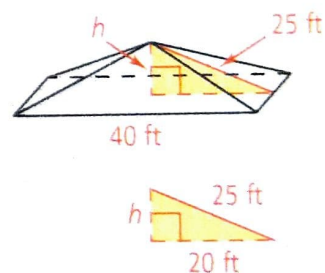
Step 2 Find the volume of the pyramid.

$V = \frac{1}{3}Bh$ Use the formula for volume of a pyramid.

$= \frac{1}{3}(40 \cdot 40)(15)$ Substitute $40 \cdot 40$ for B and 15 for h .

$= 8000$ Simplify.

The volume of the pyramid is 8000 ft^3 .

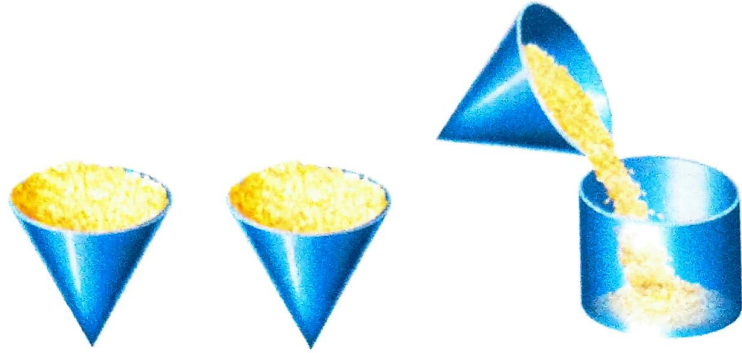


Got It? 2. What is the volume of a square pyramid with base edges 24 m and slant height 13 m?

How is this similar to finding the volume of a prism? In both cases, you need the area of the base and the height.

How do you use the slant height? The slant height is the length of the hypotenuse of the right triangle. Use the slant height to find the height of the pyramid.

Essential Understanding The volume of a cone is related to the volume of a cylinder with the same base and height.



The cones and the cylinder have the same base and height. It takes three cones full of rice to fill the cylinder.

Take note

Theorem 11-9 Volume of a Cone

The volume of a cone is one third the product of the area of the base and the height of the cone.

$$V = \frac{1}{3}Bh, \text{ or } V = \frac{1}{3}\pi r^2h$$



A cone-shaped structure can be particularly strong, as downward forces at the vertex are distributed to all points in its circular base.



Problem 3 Finding the Volume of a Cone STEM

Traditional Architecture The covering on a tepee rests on poles that come together like concurrent lines. The resulting structure approximates a cone. If the tepee pictured is 12 ft high with a base diameter of 14 ft, what is its approximate volume?

$$\begin{aligned} V &= \frac{1}{3}\pi r^2h && \text{Use the formula for the volume of a cone.} \\ &= \frac{1}{3}\pi(7)^2(12) && \text{Substitute } \frac{14}{2}, \text{ or } 7, \text{ for } r \text{ and } 12 \text{ for } h. \\ &\approx 615.7521601 && \text{Use a calculator.} \end{aligned}$$

The volume of the tepee is approximately 616 ft³.



How is this similar to finding the volume of a cylinder?

In both cases, you need to find the base area of a circle.



Got It? 3. a. The height and radius of a child's tepee are half those of the tepee in Problem 3. What is the volume of the child's tepee to the nearest cubic foot?

b. **Reasoning** What is the relationship between the volume of the original tepee and the child's tepee?

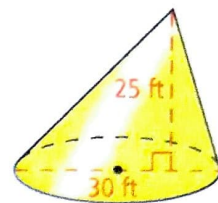
This volume formula applies to all cones, including oblique cones.

Problem 4 Finding the Volume of an Oblique Cone

What is the volume of the oblique cone at the right? Give your answer in terms of π and also rounded to the nearest cubic foot.

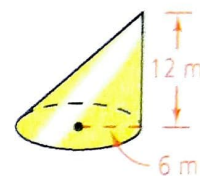
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Use the formula for volume of a cone.} \\ &= \frac{1}{3}\pi(15)^2(25) && \text{Substitute 15 for } r \text{ and 25 for } h. \\ &= 1875\pi && \text{Simplify.} \\ &\approx 5890.486225 && \text{Use a calculator.} \end{aligned}$$

The volume of the cone is $1875\pi \text{ ft}^3$, or about 5890 ft^3 .



What is the height of the oblique cone?
The height is the length of the perpendicular segment from the vertex of the cone to the base, which is 25 ft. In an oblique cone, the segment does not intersect the center of the base.

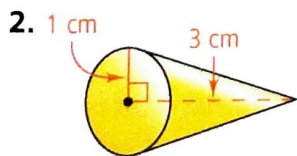
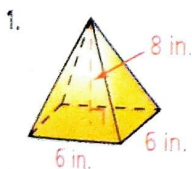
- Got It?** 4. a. What is the volume of the oblique cone at the right in terms of π and rounded to the nearest cubic meter?
 b. **Reasoning** How does the volume of an oblique cone compare to the volume of a right cone with the same diameter and height? Explain.



Lesson Check

Do you know HOW?

What is the volume of each figure? If necessary, round to the nearest tenth.



Do you UNDERSTAND? **MATHEMATICAL PRACTICES**

3. **Compare and Contrast** How are the formulas for the volume of a pyramid and the volume of a cone alike? How are they different?
4. **Error Analysis** A square pyramid has base edges 13 ft and height 10 ft. A cone has diameter 13 ft and height 10 ft. Your friend claims the figures have the same volume because the volume formulas for a pyramid and a cone are the same: $V = \frac{1}{3}Bh$. What is her error?

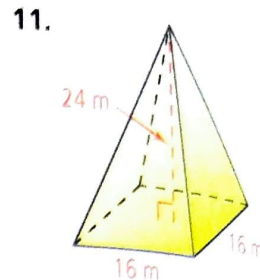
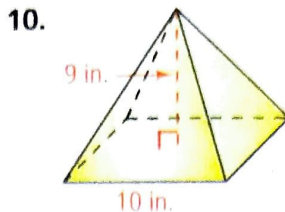
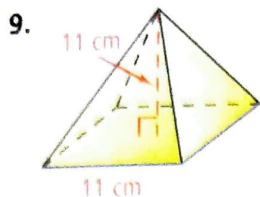
Practice and Problem-Solving Exercises **MATHEMATICAL PRACTICES**

Find the volume of each square pyramid.

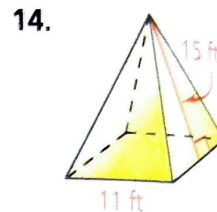
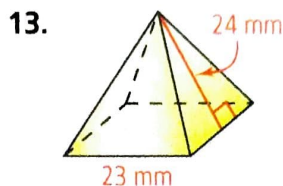
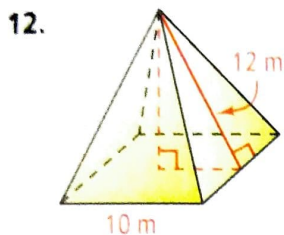
See Problem 1.

5. base edges 10 cm, height 6 cm
6. base edges 18 in., height 12 in.
7. base edges 5 m, height 6 m
8. **Buildings** The Transamerica Pyramid Building in San Francisco is 853 ft tall with a square base that is 149 ft on each side. To the nearest thousand cubic feet, what is the volume of the Transamerica Pyramid?

Find the volume of each square pyramid. Round to the nearest tenth if necessary.

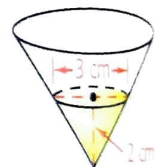


Find the volume of each square pyramid, given its slant height. Round to the nearest tenth.



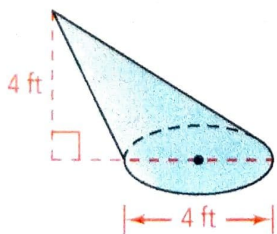
STEM 15. **Chemistry** In a chemistry lab you use a filter paper cone to filter a liquid. The diameter of the cone is 6.5 cm and its height is 6 cm. How much liquid will the cone hold when it is full?

STEM 16. **Chemistry** This cone has a filter that was being used to remove impurities from a solution but became clogged and stopped draining. The remaining solution is represented by the shaded region. How many cubic centimeters of the solution remain in the cone?

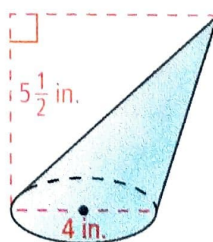


Find the volume of each cone in terms of π and also rounded as indicated.

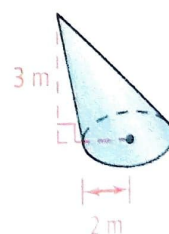
17. nearest cubic foot



18. nearest cubic inch



19. nearest cubic meter



B Apply

20. **Think About a Plan** A cone with radius 1 fits snugly inside a square pyramid, which fits snugly inside a cube. What are the volumes of the three figures?

- How can you *draw a diagram* of the situation?
- What dimensions do the cone, pyramid, and cube have in common?

21. **Reasoning** Suppose the height of a pyramid is halved. How does this affect its volume? Explain.

22. **Writing** Without doing any calculations, explain how the volume of a cylinder with $B = 5\pi \text{ cm}^2$ and $h = 20 \text{ cm}$ compares to the volume of a cone with the same base area and height.

Answers

Practice and Problem-Solving Exercises (continued)

- 9.** 443.7 cm^3
- 10.** 300 in.^3
- 11.** 2048 m^3
- 12.** 363.6 m^3
- 13.** 3714.5 mm^3
- 14.** 562.9 ft^3
- 15.** about 66.4 cm^3
- 16.** about 4.7 cm^3
- 17.** $\frac{16}{3}\pi \text{ ft}^3$; 17 ft^3
- 18.** $\frac{22}{3}\pi \text{ in.}^3$; 23 in.^3
- 19.** $4\pi \text{ m}^3$; 13 m^3