

11-6

Surface Areas and Volumes of Spheres

Common Core State Standards

G-GMD.A.3 Use volume formulas for . . . spheres to solve problems.

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects.

MP 1, MP 3, MP 4, MP 6, MP 7, MP 8

Objective To find the surface area and volume of a sphere



Try drawing a diagram of a sphere to see how different cross sections compare.



SOLVE IT!

Getting Ready!

The three orange slices below were cut from three different oranges. Do you have sufficient information to tell which orange is the largest? If not, explain what information you would need.

A

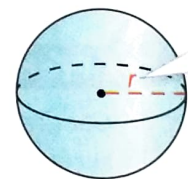
B

C

- Lesson Vocabulary**
- sphere
 - center of a sphere
 - radius of a sphere
 - diameter of a sphere
 - circumference of a sphere
 - great circle
 - hemisphere

In the Solve It, you considered the sizes of objects with circular cross sections.

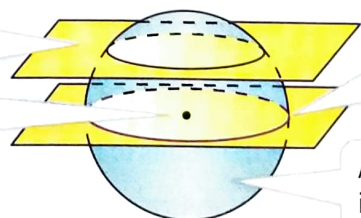
A **sphere** is the set of all points in space equidistant from a given point called the **center**. A **radius** is a segment that has one endpoint at the center and the other endpoint on the sphere. A **diameter** is a segment passing through the center with endpoints on the sphere.



r is the length of the radius of the sphere.

Essential Understanding You can find the surface area and the volume of a sphere when you know its radius.

When a plane and a sphere intersect in more than one point, the intersection is a circle. If the center of the circle is also the center of the sphere, it is called a **great circle**.



The circumference of a great circle is the **circumference** of the sphere.

A great circle divides a sphere into two **hemispheres**.

A baseball can model a sphere. To approximate its surface area, you can take apart its covering. Each of the two pieces suggests a pair of circles with radius r , which is approximately the radius of the ball. The area of the four circles, $4\pi r^2$, suggests the surface area of the ball.



The area of each circle is πr^2 .

Take note

Theorem 11-10 Surface Area of a Sphere

The surface area of a sphere is four times the product of π and the square of the radius of the sphere.

$$S.A. = 4\pi r^2$$



Problem 1 Finding the Surface Area of a Sphere

What is the surface area of the sphere in terms of π ?

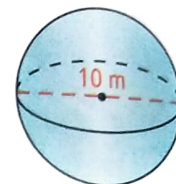
The diameter is 10 m, so the radius is $\frac{10}{2}$ m, or 5 m.

$$S.A. = 4\pi r^2 \quad \text{Use the formula for surface area of a sphere.}$$

$$= 4\pi(5)^2 \quad \text{Substitute 5 for } r.$$

$$= 100\pi \quad \text{Simplify.}$$

The surface area is $100\pi \text{ m}^2$.



Plan

What are you given?

In sphere problems, make it a habit to note whether you are given the radius or the diameter. In this case, you are given the diameter.



Got It? 1. What is the surface area of a sphere with a diameter of 14 in.? Give your answer in terms of π and rounded to the nearest square inch.

You can use spheres to approximate the surface areas of real-world objects.



Problem 2 Finding Surface Area

Geography Earth's equator is about 24,902 mi long. What is the approximate surface area of Earth? Round to the nearest thousand square miles.

Step 1 Find the radius of Earth.

$$C = 2\pi r \quad \text{Use the formula for circumference.}$$

$$24,902 = 2\pi r \quad \text{Substitute 24,902 for } C.$$

$$\frac{24,902}{2\pi} = r \quad \text{Divide each side by } 2\pi.$$

$$r \approx 3963.276393 \quad \text{Use a calculator.}$$

Step 2 Use the radius to find the surface area of Earth.

$$S.A. = 4\pi r^2 \quad \text{Use the formula for surface area.}$$

$$= 4\pi \text{ ANS } \boxed{x^2} \boxed{\text{enter}} \quad \text{Use a calculator. ANS uses the value of } r \text{ from Step 1.}$$

$$\approx 197387017.5$$

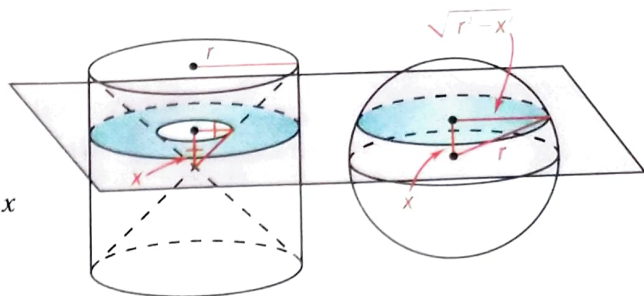
The surface area of Earth is about $197,387,000 \text{ mi}^2$.



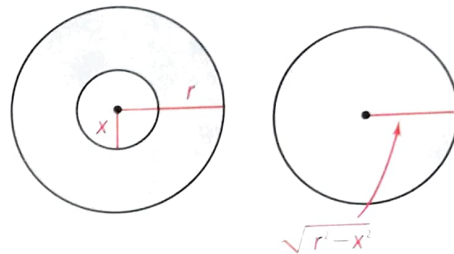
Got It? 2. What is the surface area of a melon with circumference 18 in.? Round your answer to the nearest ten square inches.

In the previous lesson, you learned that the volume of a cone is $\frac{1}{3}\pi r^3$. You can use this with Cavalieri's Principle to find the formula for the volume of a sphere.

Both figures at the right have a parallel plane x units above their centers that form circular cross sections.



The area of the cross section of the cylinder minus the area of the cross section of the cone is the same as the area of the cross section of the sphere. Every horizontal plane will cut the figures into cross sections of equal area. By Cavalieri's Principle, the volume of the sphere = the volume of the cylinder = the volume of two cones.



$$\begin{aligned} \text{Volume of a sphere} &= \pi r^2(2r) - 2\left(\frac{1}{3}\pi r^3\right) \\ &= 2\pi r^3 - \frac{2}{3}\pi r^3 \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

take note

Theorem 11-11 Volume of a Sphere

The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.

$$V = \frac{4}{3}\pi r^3$$

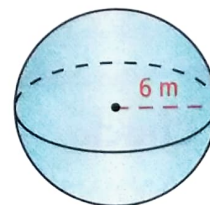


Problem 3 Finding the Volume of a Sphere

What is the volume of the sphere in terms of π ?

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 && \text{Use the formula for volume of a sphere.} \\ &= \frac{4}{3}\pi(6)^3 && \text{Substitute.} \\ &= 288\pi \end{aligned}$$

The volume of the sphere is $288\pi \text{ m}^3$.



Think
What are the units of the answer?
You are cubing the radius, which is in meters (m), so your answer should be in cubic meters (m^3).

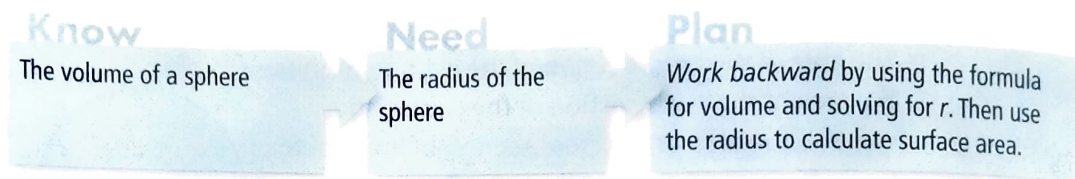


- Got It?** 3. a. A sphere has a diameter of 60 in. What is its volume to the nearest cubic inch?
 b. **Reasoning** Suppose the radius of a sphere is halved. How does this affect the volume of the sphere? Explain.

Notice that you only need to know the radius of a sphere to find its volume and surface area. This means that if you know the volume of a sphere, you can find its surface area.

Problem 4 Using Volume to Find Surface Area

The volume of a sphere is 5000 m^3 . What is its surface area to the nearest square meter?



Step 1 Find the radius of the sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Use the formula for volume of a sphere.}$$

$$5000 = \frac{4}{3}\pi r^3 \quad \text{Substitute.}$$

$$5000\left(\frac{3}{4\pi}\right) = r^3 \quad \text{Solve for } r^3.$$

$$\sqrt[3]{5000\left(\frac{3}{4\pi}\right)} = r \quad \text{Take the cube root of each side.}$$

$$r \approx 10.60784418 \quad \text{Use a calculator.}$$


Step 2 Find the surface area of the sphere.

$$\text{S.A.} = 4\pi r^2 \quad \text{Use the formula for surface area of a sphere.}$$

$$= 4\pi \text{ ANS } \left[\text{x}^2 \right] \left[\text{enter} \right] \quad \text{Use a calculator.}$$

$$\approx 1414.04792$$

The surface area of the sphere is about 1414 m^2 .

 **Got It?** 4. The volume of a sphere is 4200 ft^3 . What is its surface area to the nearest tenth?



Lesson Check

Do you know HOW?

The diameter of a sphere is 12 ft.

1. What is its surface area in terms of π ?
2. What is its volume to the nearest tenth?
3. The volume of a sphere is $80\pi \text{ cm}^3$. What is its surface area to the nearest whole number?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

-  4. **Vocabulary** What is the ratio of the area of a great circle to the surface area of the sphere?
-  5. **Error Analysis** Your classmate claims that if you double the radius of a sphere, its surface area and volume will quadruple. What is your classmate's error? Explain.

Practice

Find the surface area of the sphere with the given diameter or radius. Leave your answer in terms of π .

See Problem 1.

6. $d = 30$ m

7. $r = 10$ in.

8. $d = 32$ mm

9. $r = 100$ yd

Sports Find the surface area of each ball. Leave each answer in terms of π .

10.



$d = 68$ mm

11.



$d = 21$ cm

12.



$d = 2\frac{1}{16}$ in.

Use the given circumference to find the surface area of each spherical object. Round your answer to the nearest whole number.

See Problem 2.

13. a grapefruit with $C = 14$ cm

14. a bowling ball with $C = 27$ in.

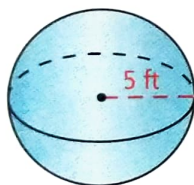
15. a pincushion with $C = 8$ cm

16. a head of lettuce with $C = 22$ in.

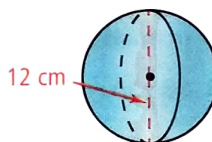
Find the volume of each sphere. Give each answer in terms of π and rounded to the nearest cubic unit.

See Problem 3.

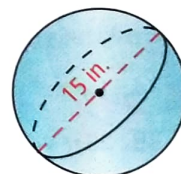
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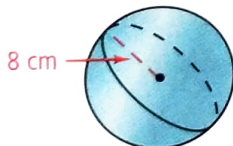
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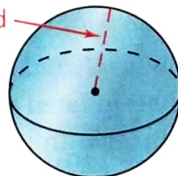
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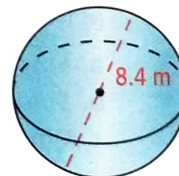
20.



21. 12 yd



22.



A sphere has the volume given. Find its surface area to the nearest whole number.

See Problem 4.

23. $V = 900$ in.³

24. $V = 3000$ m³

25. $V = 140$ cm³

Apply

26. **Mental Math** Use $\pi \approx 3$ to estimate the surface area and volume of a sphere with radius 3 cm.

27. **Open-Ended** Give the dimensions of a cylinder and a sphere that have the same volume.