11-7 Areas and Volumes of Similar Solids

Common Core State Standards

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects.

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations . . .

MP 3, MP 4, MP 7, MP 8

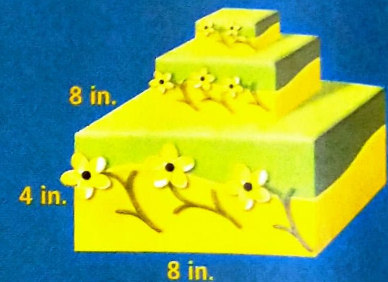
Objective To compare and find the areas and volumes of similar solids

Will the bottom-to-middle ratio be the same as the middle-to-top ratio?

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

A baker is making a three-layer wedding cake. Each layer has a square base. Each dimension of the middle layer is $\frac{1}{2}$ the corresponding dimension of the bottom layer. Each dimension of the top layer is $\frac{1}{2}$ the corresponding dimension of the middle layer. What conjecture can you make about the relationship between the volumes of the layers? Calculate the volumes to check your answer. Modify your conjecture if necessary.



Lesson Vocabulary
- similar solids

Essential Understanding

You can use ratios to compare the areas and volumes of similar solids.

Similar solids have the same shape, and all their corresponding dimensions are proportional. The ratio of corresponding linear dimensions of two similar solids is the scale factor. Any two cubes are similar, as are any two spheres.

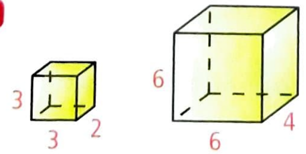
Problem 1 Identifying Similar Solids

Plan

How do you check for similarity?
Check that the ratios of the corresponding dimensions are the same. A rectangular prism has three dimensions (ℓ, w, h), so you must check three ratios.

Are the two rectangular prisms similar? If so, what is the scale factor of the first figure to the second figure?

A

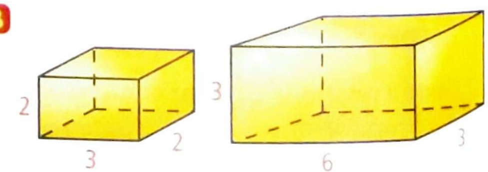


$$\frac{3}{6} = \frac{2}{4} = \frac{3}{6}$$

The prisms are similar because the corresponding linear dimensions are proportional.

The scale factor is $\frac{1}{2}$.

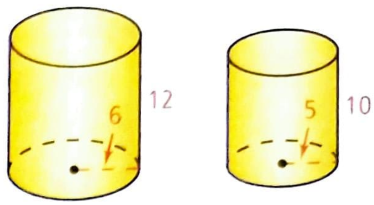
B



$$\frac{2}{3} = \frac{2}{3} \neq \frac{3}{6}$$

The prisms are not similar because the corresponding linear dimensions are not proportional.

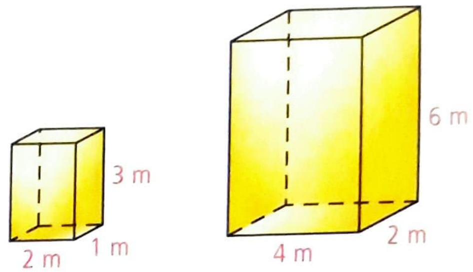
Got It? 1. Are the two cylinders similar? If so, what is the scale factor of the first figure to the second figure?



The two similar prisms shown here suggest two important relationships for similar solids.

The ratio of the side lengths is 1 : 2.
The ratio of the surface areas is 22 : 88, or 1 : 4.
The ratio of the volumes is 6 : 48, or 1 : 8.

The ratio of the surface areas is the square of the scale factor. The ratio of the volumes is the cube of the scale factor. These two facts apply to all similar solids.



S.A. = 22 m^2
$V = 6$ m^3

S.A. = 88 m^2
$V = 48$ m^3

take note

Theorem 11-12 Areas and Volumes of Similar Solids

If the scale factor of two similar solids is $a : b$, then

- the ratio of their corresponding areas is $a^2 : b^2$
- the ratio of their volumes is $a^3 : b^3$

Plan

ow can you use the iven information?

ou are given the olumes of two similar lids. Write a proportion ing the ratio $a^3 : b^3$.

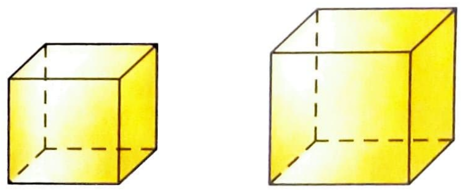
Problem 2 Finding the Scale Factor

The square prisms at the right are similar. What is the scale factor of the smaller prism to the larger prism?

$\frac{a^3}{b^3} = \frac{729}{1331}$ The ratio of the volumes is $a^3 : b^3$.

$\frac{a}{b} = \frac{9}{11}$ Take the cube root of each side.

The scale factor is 9 : 11.



$V = 729$ cm^3

$V = 1331$ cm^3

Got It? 2. a. What is the scale factor of two similar prisms with surface areas 144 m^2 and 324 m^2?

b. Reasoning Are any two square prisms similar? Explain.

Problem 3 Using a Scale Factor

Painting The lateral areas of two similar paint cans are 1019 cm^2 and 425 cm^2. The volume of the smaller can is 1157 cm^3. What is the volume of the larger can?

Know

- The lateral areas
- The volume of the smaller can

Need

The scale factor

Plan

Use the lateral areas to find the scale factor $a : b$. Then write and solve a proportion using the ratio $a^3 : b^3$ to find the volume of the larger can.

Step 1 Find the scale factor $a : b$.

$$\frac{a^2}{b^2} = \frac{1019}{425}$$ The ratio of the surface areas is $a^2 : b^2$.

$$\frac{a}{b} = \frac{\sqrt{1019}}{\sqrt{425}}$$ Take the positive square root of each side.

Think

Does it matter how you set up the proportion?
Yes. The numerators should refer to the same paint can, and the denominators should refer to the other can.

Step 2 Use the scale factor to find the volume.

$$\frac{V_{\text{large}}}{V_{\text{small}}} = \frac{\left(\sqrt{1019}\right)^3}{\left(\sqrt{425}\right)^3}$$ The ratio of the volumes is $a^3 : b^3$.

$$\frac{V_{\text{large}}}{1157} = \frac{\left(\sqrt{1019}\right)^3}{\left(\sqrt{425}\right)^3}$$ Substitute 1157 for V_{small}.

$$V_{\text{large}} = 1157 \cdot \frac{\left(\sqrt{1019}\right)^3}{\left(\sqrt{425}\right)^3}$$ Solve for V_{large}.

$$V_{\text{large}} \approx 4295.475437$$ Use a calculator.

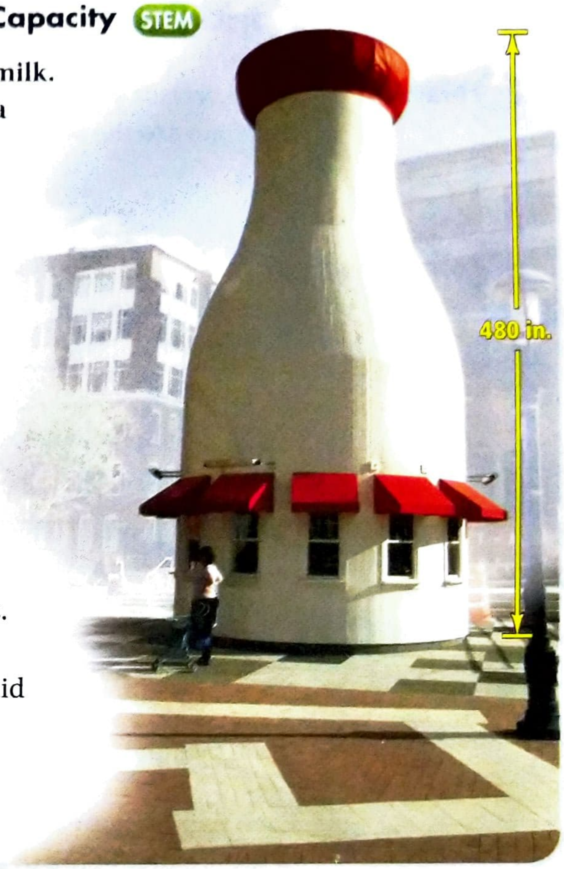
The volume of the larger paint can is about 4295 cm^3.

Got It? **3.** The volumes of two similar solids are 128 m^3 and 250 m^3. The surface area of the larger solid is 250 m^2. What is the surface area of the smaller solid?

You can compare the capacities and weights of similar objects. The capacity of an object is the amount of fluid the object can hold. The capacities and weights of similar objects made of the same material are proportional to their volumes.

Problem 4 Using a Scale Factor to Find Capacity STEM

Containers **A bottle that is 10 in. high holds 34 oz of milk. The sandwich shop shown at the right is shaped like a milk bottle. To the nearest thousand ounces how much milk could the building hold?**



Think

How does capacity relate to volume?
Since the capacities of similar objects are proportional to their volumes, the ratio of their capacities is equal to the ratio of their volumes.

The scale factor of the bottles is 1 : 48.

The ratio of their volumes, and hence the ratio of their capacities, is $1^3 : 48^3$, or 1 : 110,592.

$\frac{1}{110{,}592} = \frac{34}{x}$ Let x = the capacity of the milk-bottle building.

$x = 34 \cdot 110{,}592$ Use the Cross Products Property.

$x = 3{,}760{,}128$ Simplify.

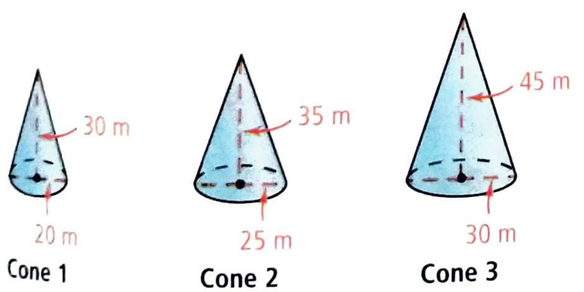
The milk-bottle building could hold about 3,760,000 oz.

Got It? 4. A marble paperweight shaped like a pyramid weighs 0.15 lb. How much does a similarly shaped marble paperweight weigh if each dimension is three times as large?

Lesson Check

Do you know HOW?

1. Which two of the following cones are similar? What is their scale factor?



2. The volumes of two similar containers are 115 in.^{3} and 67 in.^{3}. The surface area of the smaller container is 108 in.^{2}. What is the surface area of the larger container?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. **Vocabulary** How are similar solids different from similar polygons? Explain.

4. **Error Analysis** Two cubes have surface areas 49 cm^2 and 64 cm^2. Your classmate tried to find the scale factor of the larger cube to the smaller cube. Explain and correct your classmate's error.

$\frac{a^2}{b^2} = \frac{49}{64}$

$\frac{a}{b} = \frac{7}{8}$

The scale factor of the larger cube to the smaller cube is 7 : 8.

Practice and Problem-Solving Exercises

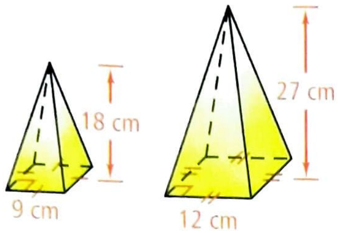
MATHEMATICAL PRACTICES

A Practice

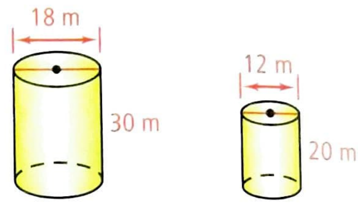
For Exercises 5–10, are the two figures similar? If so, give the scale factor of the first figure to the second figure.

See Problem 1.

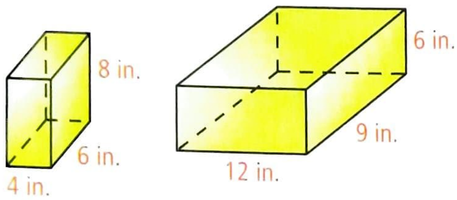
5.



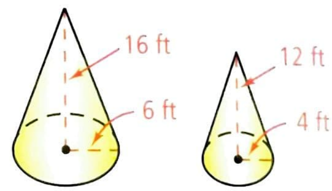
6.



7.



8.



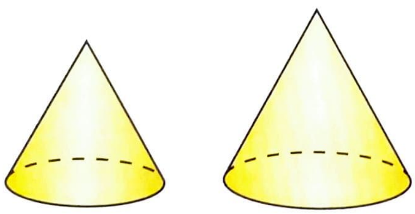
9. two cubes, one with 3-cm edges, the other with 4.5-cm edges

10. a cylinder and a square prism both with 3-in. radius and 1-in. height

Each pair of figures is similar. Use the given information to find the scale factor of the smaller figure to the larger figure.

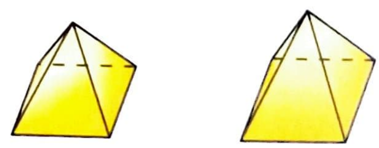
See Problem 2.

11.



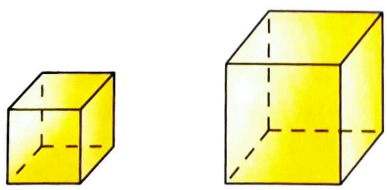
$V = 250\pi$ ft^3 $V = 432\pi$ ft^3

12.



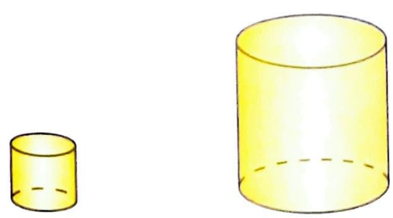
$V = 216$ in.^{3} $V = 343$ in.^{3}

13.



S.A. = 18 m^2 S.A. = 32 m^2

14.



S.A. = 20π yd^2 S.A. = 125π yd^2

The surface areas of two similar figures are given. The volume of the larger figure is given. Find the volume of the smaller figure.

See Problem 3.

15. S.A. = 248 in.^{2}
S.A. = 558 in.^{2}
$V = 810$ in.^{3}

16. S.A. = 192 m^2
S.A. = 1728 m^2
$V = 4860$ m^3

17. S.A. = 52 ft^2
S.A. = 208 ft^2
$V = 192$ ft^3