10.1 The Square Root Property

1 Know that every positive real number has two square roots.

2 Solve quadratic equations using the square root property.

In Section 5.6 we solved quadratic equations by factoring. Recall that **quadratic equations** are equations of the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, $a \neq 0$. A quadratic equation in this form is said to be in **standard form**. Solving quadratic equations by factoring is the preferred technique when the factors can be found quickly.

However, not every quadratic equation can be factored easily, and many cannot be factored at all. In this chapter we give two techniques, completing the square and the quadratic formula, for solving quadratic equations that cannot be solved by factoring.

1 Know That Every Positive Real Number Has Two Square Roots

In Section 9.1 we stated that every positive number has two square roots. For example, the positive or principal square root of 49 is 7.

$$\sqrt{49} = 7$$

The negative square root of 49 is -7.

$$-\sqrt{49} = -7$$

Understanding Algebra

The symbol $\pm$ is read "plus or minus."

The two square roots of 49 are $+7$ and -7. A convenient way to indicate the two square roots of a number is to use the plus or minus symbol, $\pm$. For example, the square roots of 49 can be indicated ± 7, read "plus or minus 7."

Number	Both Square Roots
64	± 8
100	± 10
3	$\pm\sqrt{3}$
7	$\pm\sqrt{7}$

An approximation of a number like $-\sqrt{5}$ can be found by evaluating $\sqrt{5}$ on your calculator and then taking its opposite or negative value.

$$\sqrt{5} \approx 2.24 \quad \text{(rounded to the nearest hundredth)}$$
$$-\sqrt{5} \approx -2.24$$

Now consider the equation

$$x^2 = 49$$

We can see by substitution that this equation has two solutions, 7 and -7.

Check

$x = 7$	$x = -7$
$x^2 = 49$	$x^2 = 49$
$7^2 \stackrel{?}{=} 49$	$(-7)^2 \stackrel{?}{=} 49$
$49 = 49$ True	$49 = 49$ True

Therefore, the solutions to the equation $x^2 = 49$ are 7 and -7 (or ± 7).

2 Solve Quadratic Equations Using the Square Root Property

In general, for any quadratic equation of the form $x^2 = a$, we can use the **square root property** to obtain the solution.

Square Root Property

If $x^2 = a$, then $x = \sqrt{a}$ or $x = -\sqrt{a}$ (abbreviated $x = \pm\sqrt{a}$).

For example, if $x^2 = 7$, then by the square root property, $x = \sqrt{7}$ or $x = -\sqrt{7}$. We may also write $x = \pm\sqrt{7}$.

EXAMPLE 1 Solve the equation $x^2 - 25 = 0$.

Solution Before we use the square root property we must isolate the squared variable. Add 25 to both sides of the equation to get the variable by itself on one side of the equation.

$$x^2 - 25 = 0$$

$$x^2 = 25 \quad \text{Added 25 to both sides}$$

$$x = \pm\sqrt{25} \quad \text{Used square root property.}$$

$$x = \pm 5$$

Check in the original equation.

Check

$x = 5$	$x = -5$
$x^2 - 25 = 0$	$x^2 - 25 = 0$
$5^2 - 25 \stackrel{?}{=} 0$	$(-5)^2 - 25 = 0$
$25 - 25 \stackrel{?}{=} 0$	$25 - 25 \stackrel{?}{=} 0$
$0 = 0$ True	$0 = 0$ True

Now Try Exercise 7

Understanding Algebra

When solving $x^2 = 25$, be sure to take *both square roots* to get

$x = \pm\sqrt{25} = \pm 5$

EXAMPLE 2 Solve the equation $x^2 + 10 = 74$.

Solution

$$x^2 + 10 = 74$$

$$x^2 = 64 \quad \text{Subtracted 10 from both sides}$$

$$x = \pm\sqrt{64} \quad \text{Square root property}$$

$$x = \pm 8$$

Now Try Exercise 13

EXAMPLE 3 Solve the equation $a^2 - 13 = 0$.

Solution

$$a^2 - 13 = 0$$

$$a^2 = 13 \quad \text{Added 13 to both sides}$$

$$a = \pm\sqrt{13} \quad \text{Square root property}$$

Now Try Exercise 15

Understanding Algebra

In Example 3, the *exact* solutions are $\pm\sqrt{13}$. The *approximate* solutions are ± 3.60555.

EXAMPLE 4 Solve the equation $(x - 3)^2 = 4$.

Solution Begin by using the square root property.

$$(x - 3)^2 = 4$$

$$x - 3 = \pm\sqrt{4} \quad \text{Square root property}$$

$$x - 3 = \pm 2$$

$$x - 3 + 3 = 3 \pm 2 \quad \text{Add 3 to both sides.}$$

$$x = 3 \pm 2$$

$$x = 3 + 2 \quad \text{or} \quad x = 3 - 2$$

$$x = 5 \qquad\qquad x = 1$$

The solutions are 1 and 5.

Now Try Exercise 27

EXAMPLE 5 Solve the equation $(5x + 4)^2 - 2 = 16$.

Solution We must first isolate the squared term.

$$(5x + 4)^2 - 2 = 16$$
$$(5x + 4)^2 = 18 \quad \text{Added 2 to both sides to isolate the squared term.}$$
$$5x + 4 = \pm\sqrt{18} \quad \text{Square root property}$$
$$5x + 4 = \pm\sqrt{9}\,\sqrt{2} \quad \text{Simplify } \sqrt{18}.$$
$$5x + 4 = \pm 3\sqrt{2}$$
$$5x + 4 - 4 = -4 \pm 3\sqrt{2} \quad \text{Subtract 4 from both sides.}$$
$$5x = -4 \pm 3\sqrt{2}$$
$$x = \frac{-4 \pm 3\sqrt{2}}{5} \quad \text{Divide both sides by 5.}$$

The solutions are $\frac{-4 + 3\sqrt{2}}{5}$ and $\frac{-4 - 3\sqrt{2}}{5}$.

Now Try Exercise 43

Now let's look at one of many applications of quadratic equations.

EXAMPLE 6 **Creating Advertisements** Antoinette LeMans designed a magazine advertisement for her company in the shape of a rectangle whose length is 1.62 times its width. Find the dimensions of the advertisement if it is to have an area of 20 square inches. See **Figure 10.1**.

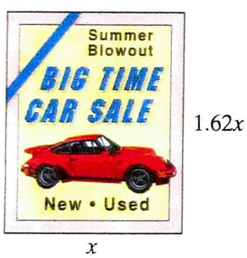

FIGURE 10.1

Solution Understand and Translate

Let x = width of rectangle.

Then $1.62x$ = length of rectangle.

$$\text{area} = \text{length} \cdot \text{width}$$
$$20 = (1.62x)x$$

Carry Out

$$20 = 1.62x^2$$
$$\text{or} \quad 1.62x^2 = 20$$
$$x^2 = \frac{20}{1.62} \approx 12.3$$
$$x \approx \pm\sqrt{12.3} \approx \pm 3.51 \text{ inches}$$

Check and Answer Since the width cannot be negative, the width, x, is approximately 3.51 inches. The length is about $1.62(3.51) = 5.69$ inches.

Check

$$\text{area} = \text{length} \cdot \text{width}$$
$$20 \stackrel{?}{=} (5.69)(3.51)$$
$$20 \approx 19.97 \quad \text{True}$$

(There is a slight round-off error due to rounding off decimal answers.)

Now Try Exercise 57

EXERCISE SET 10.1

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Warm-Up Exercises

Fill in the blanks with the appropriate word, phrase, or symbol(s) from the following list.

three	standard	one	square root property	$\pm\sqrt{10}$
add	two	square	isolate	quadratic

1. A ____________ equation is an equation that can be written as $ax^2 + bx + c = 0$.

2. An equation written in the form $ax^2 + bx + c = 0$ is said to be in ____________ form.