10.3 Solving Quadratic Equations by the Quadratic Formula

1. Solve quadratic equations by the quadratic formula.
2. Determine the number of solutions to a quadratic equation using the discriminant.

Understanding Algebra

The standard form of a quadratic equation is $ax^2 + bx + c = 0$.

1 Solve Quadratic Equations by the Quadratic Formula

Another method that can be used to solve any quadratic equation is the **quadratic formula**. It is the most versatile method of solving quadratic equations.

Quadratic Equation in Standard Form	Values of a, b, and c		
$x^2 - 5x + 6 = 0$	$a = 1$,	$b = -5$,	$c = 6$
$5x^2 + 3x = 0$	$a = 5$,	$b = 3$,	$c = 0$
$-\frac{1}{2}x^2 + 5 = 0$	$a = -\frac{1}{2}$,	$b = 0$,	$c = 5$

We can develop the quadratic formula by starting with a quadratic equation in standard form and completing the square, as discussed in the preceding section.

$$ax^2 + bx + c = 0 \qquad \text{Standard form of quadratic equation}$$

$$\frac{ax^2}{a} + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{Divide both sides by } a.$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \frac{c}{a} \text{ was subtracted from both sides.}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \qquad \text{Take } \frac{1}{2} \text{ of } \frac{b}{a}\text{; and square it to get } \frac{b^2}{4a^2}\text{. Then add this expression to both sides.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} \qquad \text{Rewrite the left side of the equation as the square of a binomial.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{Write the right side with a common denominator.}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{Square root property}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Quotient rule for radicals, } \sqrt{4a^2} = 2a$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \qquad \frac{b}{2a} \text{ was subtracted from both sides.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Write with a common denominator to get the quadratic formula.}$$

To Solve a Quadratic Equation by the Quadratic Formula

1. Write the equation in standard form, $ax^2 + bx + c = 0$, and determine the numerical values for a, b, and c.
2. Substitute the values for a, b, and c from step 1 into the quadratic formula below and then evaluate to obtain the solution.

THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 1 Use the quadratic formula to solve the equation $x^2 + 4x + 3 = 0$.

Solution In this equation $a = 1$, $b = 4$, and $c = 3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(3)}}{2(1)}$$ Substitute values for a, b, and c.

$$= \frac{-4 \pm \sqrt{16 - 12}}{2}$$ Evaluate.

$$= \frac{-4 \pm \sqrt{4}}{2}$$

$$= \frac{-4 \pm 2}{2}$$

$$x = \frac{-4 + 2}{2} \quad \text{or} \quad x = \frac{-4 - 2}{2}$$

$$= \frac{-2}{2} = -1 \qquad = \frac{-6}{2} = -3$$

Understanding Algebra

Always recognize the values of a, b, and c before using them in the quadratic formula.

Check

$x = -1$

$$x^2 + 4x + 3 = 0$$
$$(-1)^2 + 4(-1) + 3 \stackrel{?}{=} 0$$
$$1 - 4 + 3 \stackrel{?}{=} 0$$
$$0 = 0 \quad \text{True}$$

$x = -3$

$$x^2 + 4x + 3 = 0$$
$$(-3)^2 + 4(-3) + 3 \stackrel{?}{=} 0$$
$$9 - 12 + 3 \stackrel{?}{=} 0$$
$$0 = 0 \quad \text{True}$$

Now Try Exercise 29

Avoiding Common Errors

The *entire numerator* of the quadratic formula must be divided by $2a$.

CORRECT	INCORRECT
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	~~$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$~~ ~~$x = \frac{-b}{2a} \pm \sqrt{b^2 - 4ac}$~~

EXAMPLE 2 Use the quadratic formula to solve the equation $8x^2 + 2x - 1 = 0$.

Solution

$$8x^2 + 2x - 1 = 0$$

$$a = 8, \quad b = 2, \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(2) \pm \sqrt{(2)^2 - 4(8)(-1)}}{2(8)}$$

$$= \frac{-2 \pm \sqrt{4 + 32}}{16}$$

$$= \frac{-2 \pm \sqrt{36}}{16}$$

$$= \frac{-2 \pm 6}{16}$$

$$x = \frac{-2 + 6}{16} \quad \text{or} \quad x = \frac{-2 - 6}{16}$$

$$= \frac{4}{16} = \frac{1}{4} \qquad = \frac{-8}{16} = -\frac{1}{2}$$

Check

$x = \frac{1}{4}$

$$8x^2 + 2x - 1 = 0$$
$$8\left(\frac{1}{4}\right)^2 + 2\left(\frac{1}{4}\right) - 1 \stackrel{?}{=} 0$$
$$8\left(\frac{1}{16}\right) + \frac{1}{2} - 1 \stackrel{?}{=} 0$$
$$\frac{1}{2} + \frac{1}{2} - 1 \stackrel{?}{=} 0$$
$$0 = 0 \quad \text{True}$$

$x = -\frac{1}{2}$

$$8x^2 + 2x - 1 = 0$$
$$8\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 1 \stackrel{?}{=} 0$$
$$8\left(\frac{1}{4}\right) - 1 - 1 \stackrel{?}{=} 0$$
$$2 - 1 - 1 \stackrel{?}{=} 0$$
$$0 = 0 \quad \text{True}$$

Now Try Exercise 43

Helpful Hint

- Be sure you learn the quadratic formula, as it will be used to solve many problems and applications in algebra and in math courses beyond algebra.
- Always recognize the values of a, b, and c before using them in the quadratic formula.

EXAMPLE 3 Use the quadratic formula to solve the equation $2w^2 + 6w - 3 = 0$.

Solution The variable in this equation is w. The procedure to solve the equation is the same.

$$a = 2, \quad b = 6, \quad c = -3$$
$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-3)}}{2(2)}$$
$$= \frac{-6 \pm \sqrt{36 + 24}}{4}$$
$$= \frac{-6 \pm \sqrt{60}}{4}$$
$$= \frac{-6 \pm \sqrt{4}\sqrt{15}}{4}$$
$$= \frac{-6 \pm 2\sqrt{15}}{4}$$

Now factor out 2 from both terms in the numerator; then divide out common factors as explained in Section 9.4.

$$w = \frac{\overset{1}{\cancel{2}}(-3 \pm \sqrt{15})}{\underset{2}{\cancel{4}}} \qquad \text{Factor. Divide out common factors.}$$
$$w = \frac{-3 \pm \sqrt{15}}{2}$$

Thus, the solutions are $w = \frac{-3 + \sqrt{15}}{2}$ and $w = \frac{-3 - \sqrt{15}}{2}$.

Now Try Exercise 49

Now let's try two examples where the equation is not in standard form.

EXAMPLE 4 Use the quadratic formula to solve the equation $x^2 = 6x - 4$.

Solution First write the equation in standard form.

$$x^2 - 6x + 4 = 0 \qquad \text{Set one side of the equation equal to zero.}$$

$$a = 1, \qquad b = -6, \qquad c = 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} \qquad \text{Substitute.}$$

$$= \frac{6 \pm \sqrt{36 - 16}}{2} \qquad \text{Simplify.}$$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$$= \frac{6 \pm \sqrt{4}\sqrt{5}}{2} \qquad \text{Product rule}$$

$$= \frac{6 \pm 2\sqrt{5}}{2}$$

$$= \frac{\overset{1}{\cancel{2}}(3 \pm \sqrt{5})}{\underset{1}{\cancel{2}}} \qquad \text{Factor out 2.}$$

$$= 3 \pm \sqrt{5}$$

The solutions are $x = 3 + \sqrt{5}$ and $x = 3 - \sqrt{5}$.

Now Try Exercise 41

Avoiding Common Errors

Many students solve quadratic equations correctly until the last step, when they make an error. Do not make the mistake of trying to simplify an answer that cannot be simplified any further. The following are answers that cannot be simplified, along with some common errors.

ANSWERS THAT CANNOT BE SIMPLIFIED	INCORRECT
$\frac{3 + 2\sqrt{5}}{2}$	~~$\frac{3 + 2\sqrt{5}}{2} = \frac{3 + \overset{1}{\cancel{2}}\sqrt{5}}{\underset{1}{\cancel{2}}} = 3 + \sqrt{5}$~~
$\frac{4 + 3\sqrt{5}}{2}$	~~$\frac{\overset{2}{\cancel{4}} + 3\sqrt{5}}{\underset{1}{\cancel{2}}} = 2 + 3\sqrt{5}$~~

EXAMPLE 5 Use the quadratic formula to solve the equation $t^2 = 36$.

Solution First write the equation in standard form.

$$t^2 - 36 = 0 \qquad \text{Set one side of the equation equal to 0.}$$

$$a = 1, \qquad b = 0, \qquad c = -36$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0 \pm \sqrt{0^2 - 4(1)(-36)}}{2(1)} \qquad \text{Substitute.}$$

$$= \frac{\pm\sqrt{144}}{2} = \frac{\pm 12}{2} = \pm 6 \qquad \text{Simplify and solve for } t.$$

Thus, the solutions are 6 and -6.

Now Try Exercise 33

The solution to Example 5 could have been solved more quickly by factoring or by using the square root property. We worked Example 5 using the quadratic formula to give you more practice using the formula.

The next example illustrates a quadratic equation that has no real number solution.

EXAMPLE 6 Use the quadratic formula to solve the equation $3x^2 = x - 1$.

Solution

$$3x^2 - x + 1 = 0$$

$$a = 3, \qquad b = -1, \qquad c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{1 - 12}}{6}$$

$$= \frac{1 \pm \sqrt{-11}}{6}$$

Since $\sqrt{-11}$ is not a real number, we stop here. This equation has no real number solution. *When given a problem of this type, your answer should be "no real number solution."*

Now Try Exercise 47

2 Determine the Number of Solutions to a Quadratic Equation Using the Discriminant

Discriminant

The expression under the square root sign in the quadratic formula is called the **discriminant**.

$$\underbrace{b^2 - 4ac}_{\text{Discriminant}}$$

The discriminant can be used to determine the number of real solutions to a quadratic equation, as shown below.

When the Discriminant Is:

1. **Greater than zero,** $b^2 - 4ac > 0$, the quadratic equation has *two distinct real number solutions*.
2. **Equal to zero,** $b^2 - 4ac = 0$, the quadratic equation has *one real number solution*.
3. **Less than zero,** $b^2 - 4ac < 0$, the quadratic equation has *no real number solution*.

We indicate this information in a shortened form in the chart below.

If $b^2 - 4ac$ is	Then the number of solutions is
Positive	Two distinct real number solutions
0	One real number solution
Negative	No real number solution

EXAMPLE 7

a) Find the discriminant of the equation $x^2 - 12x + 36 = 0$.
b) Use the discriminant to determine the number of solutions to the equation.
c) Use the quadratic formula to find the solutions, if any exist.

Solution

a) $a = 1, \quad b = -12, \quad c = 36$

$$b^2 - 4ac = (-12)^2 - 4(1)(36) = 144 - 144 = 0$$

b) Since the discriminant is equal to zero, there is one real number solution.

c)
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-12) \pm \sqrt{0}}{2(1)}$$
$$= \frac{12 \pm 0}{2} = \frac{12}{2} = 6$$

The only solution is 6.

Now Try Exercise 9

EXAMPLE 8

Without actually finding the solutions, determine whether the following equations have two distinct real number solutions, one real number solution, or no real number solution.

a) $4x^2 - 4x + 1 = 0$ **b)** $2x^2 + 13x = -10$ **c)** $6p^2 = -5p - 3$

Solution

We use the discriminant of the quadratic formula to answer these questions.

a) $b^2 - 4ac = (-4)^2 - 4(4)(1) = 16 - 16 = 0$

Since the discriminant is equal to zero, this equation has one real number solution.

b) First, rewrite $2x^2 + 13x = -10$ as $2x^2 + 13x + 10 = 0$.

$$b^2 - 4ac = (13)^2 - 4(2)(10) = 169 - 80 = 89$$

Since the discriminant is positive, this equation has two distinct real number solutions.

c) First rewrite $6p^2 = -5p - 3$ as $6p^2 + 5p + 3 = 0$

$$b^2 - 4ac = (5)^2 - 4(6)(3) = 25 - 72 = -47$$

Since the discriminant is negative, this equation has no real number solution.

Now Try Exercise 19

Many applications may be solved using the quadratic formula.

EXAMPLE 9 **Building a Border**

The Johnsons have a rectangular swimming pool that measures 30 feet by 16 feet. They want to add a concrete border of uniform width around all sides of the pool. How wide can they make the border if they want the area of the border to be 200 square feet?

Solution

Let's make a diagram of the pool; see **Figure 10.2**. Let x = uniform width of the border. Then the total length of the pool and border is $2x + 30$. The total width of the pool and border is $2x + 16$. The area of the border can be found by subtracting the area of the pool (the smaller rectangle area) from the area of the pool and border (the larger rectangle area).

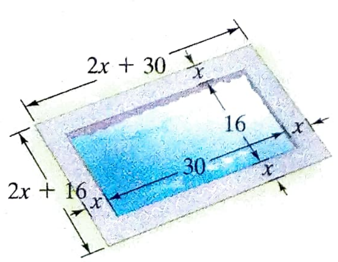

FIGURE 10.2

$$\text{area of pool} = l \cdot w = (30)(16) = 480$$
$$\text{area of pool and border} = l \cdot w = (2x + 30)(2x + 16)$$
$$= 4x^2 + 92x + 480$$

$$\text{area of border} = \text{area of pool and border} - \text{area of pool}$$
$$= (4x^2 + 92x + 480) - 480$$
$$= 4x^2 + 92x$$

The total area of the border is 200 square feet. Therefore,

$$\text{area of border} = 4x^2 + 92x$$
$$200 = 4x^2 + 92x$$

or $4x^2 + 92x - 200 = 0$ Write equation in standard form.

$4(x^2 + 23x - 50) = 0$ Factor out 4.

$\frac{1}{4} \cdot 4(x^2 + 23x - 50) = \frac{1}{4} \cdot 0$ Multiply both sides by $\frac{1}{4}$ to eliminate 4.

$x^2 + 23x - 50 = 0$

Now use the quadratic formula.

$$a = 1 \qquad b = 23 \qquad c = -50$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-23 \pm \sqrt{(23)^2 - 4(1)(-50)}}{2(1)}$$
$$= \frac{-23 \pm \sqrt{529 + 200}}{2}$$
$$= \frac{-23 \pm \sqrt{729}}{2}$$
$$= \frac{-23 \pm 27}{2}$$
$$x = \frac{-23 - 27}{2} \quad \text{or} \quad x = \frac{-23 + 27}{2}$$
$$= \frac{-50}{2} \qquad\qquad = \frac{4}{2}$$
$$= -25 \qquad\qquad = 2$$

Since lengths are positive, the only possible answer is $x = 2$. The uniform concrete border will be 2 feet wide all around the pool.

Now Try Exercise 63

Many times when working with quadratic application problems the answer is an irrational number. When this occurs in the exercise set we will round answers to two decimal places.

Helpful Hint

If all the terms in a quadratic equation have a common factor, it is easier to factor it out first so that you will have smaller numbers when you use the quadratic formula. Consider the quadratic equation $4x^2 + 8x - 12 = 0$.

In this equation $a = 4$, $b = 8$, and $c = -12$. If you solve this equation with the quadratic formula, after simplification you will get the solutions -3 and 1. Try this and see. If you factor out 4 to get

$$4x^2 + 8x - 12 = 0$$
$$4(x^2 + 2x - 3) = 0$$

and then use the quadratic formula with the equation $x^2 + 2x - 3 = 0$, where $a = 1$, $b = 2$, and $c = -3$, you get the same solution. Try this and see.

EXERCISE SET 10.3

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Warm-Up Exercises

Fill in the blanks with the appropriate word, phrase, or symbol(s) from the following list.

standard	-8	discriminant	7	-5	quadratic	add -3
two	0	add -4	one	9	add 4	

1. The __________ formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. The equation $ax^2 + bx + c = 0$ is said to be written in __________ form.

3. In the equation $7x^2 + 2x - 8 = 0$, the value of a is __________.

4. In the equation $-5x^2 + 9 = 0$, the value of b is __________.

5. The expression $b^2 - 4ac$ is called the __________.

6. If $b^2 - 4ac > 0$, then the quadratic equation has __________ real solution(s).

7. If $b^2 - 4ac = 0$, then the quadratic equation has __________ real solution(s).

8. The first step in solving $6x^2 - 5x = -4$ by the quadratic formula is to __________ to both sides.

Practice the Skills

Determine whether each equation has two distinct real number solutions, one real number solution, or no real number solution.

9. $x^2 + 5x - 9 = 0$
10. $x^2 + 2x - 7 = 0$
11. $2x^2 + x + 1 = 0$
12. $r^2 + 3r + 5 = 0$
13. $3n^2 + 2n - 5 = 0$
14. $2x^2 - 12x = -18$
15. $2m^2 - 16m = -32$
16. $5x^2 - 4x = 3$
17. $2x^2 - 7x + 10 = 0$
18. $z^2 = 5z + 11$
19. $4x = 8 + x^2$
20. $5x - 8 = 3x^2$
21. $x^2 + 7x - 2 = 0$
22. $2x^2 - 6x + 5 = 0$
23. $2.1x^2 - 0.5 = 0$
24. $0.6x^2 - 1.3x = 0$
25. $18 = -2t^2 + 12t$
26. $-16 = w^2 + 8w$

Use the quadratic formula to solve each equation. If the equation has no real number solution, so state.

27. $x^2 - 10x + 24 = 0$
28. $x^2 - 10x + 9 = 0$
29. $x^2 + 9x + 18 = 0$
30. $x^2 - 3x - 10 = 0$
31. $m^2 - 8m = -15$
32. $x^2 + 5x - 24 = 0$
33. $x^2 - 49 = 0$
34. $m^2 = 2m + 24$
35. $x^2 - 7x = 0$
36. $t^2 - 9t = 0$
37. $30 = -z^2 - 11z$
38. $z^2 - 14z + 40 = 0$
39. $x^2 - 7x - 8 = 0$
40. $n^2 - 7n + 10 = 0$
41. $2y^2 - 7y + 4 = 0$
42. $3x^2 + 5x + 1 = 0$
43. $6x^2 = -x + 1$
44. $8p^2 + 10p - 3 = 0$
45. $2x^2 = 5x + 7$
46. $4r^2 - 5 = r$
47. $2s^2 - 4s + 5 = 0$
48. $3w^2 + 2 = 4w$
49. $x^2 - 7x + 3 = 0$
50. $x^2 - 3x - 1 = 0$
51. $2x^2 - 7x = 9$
52. $-3x^2 + 17x - 20 = 0$
53. $-x^2 + 2x + 15 = 0$
54. $36 = -4s^2 + 40s$
55. $2t^2 - 6t - 56 = 0$
56. $2r^2 - 18r + 36 = 0$
57. $6y^2 + 9 = -5y$
58. $15 = -5a^2 - 5a$

Problem Solving

59. **Product of Integers** The product of two consecutive positive integers is 56. Find the two consecutive integers.

60. **Dimensions of Rectangle** The length of a rectangle is 6 feet longer than its width. Find the dimensions of the rectangle if its area is 55 square feet.

61. **Dimensions of Rectangle** The length of a rectangle is 3 feet smaller than twice its width. Find the length and width of the rectangle if its area is 20 square feet.

62. **Rectangular Garden** Sean McDonald's rectangular garden measures 20 feet by 30 feet. He wishes to build a uniform-width brick walkway around his garden that covers an area of 336 square feet. What will be the width of the walkway?

63. **Swimming Pool** Harold Goldstein and his wife Elaine recently installed a built-in rectangular swimming pool measuring 25 feet by 35 feet. They want to add a decorative tile border of uniform width around all sides of the pool. How wide can they make the tile border if they purchased enough tile to cover 256 square feet?

64. **Pottery Studio** Julie Bonds is planning to plant a grass lawn of uniform width around her rectangular pottery studio,

which measures 48 feet by 36 feet. How far will the lawn extend from the studio if Julie has only enough seed to plant 4000 square feet of grass?


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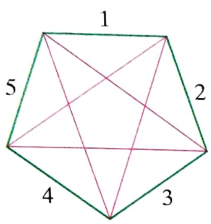
65. Diagonals in a Polygon The number of diagonals, d, in a polygon with n sides is given by the formula $d = \dfrac{n^2 - 3n}{2}$. For example, a pentagon, a polygon with 5 sides, has $d = \dfrac{5^2 - 15}{2} = 5$ diagonals; see the figure.



If a polygon has 14 diagonals, how many sides does it have?

66. Diagonals in a Polygon If a polygon has 20 diagonals, how many sides does it have? See Exercise 65.

67. Flags The cost, c, for manufacturing x American flags is given by $c = x^2 - 16x + 40$. Find the number of flags manufactured if the cost is \$120.

68. Manufacturing Cost Repeat Exercise 67 for a cost of \$2680.

69. Model Rocket Phil Chefetz launches a model rocket from the ground. The height, s, of the rocket above the ground at time t seconds after it is launched can be found by the formula $s = -16t^2 + 90t$. Find how long it will take for the rocket to reach a height of 80 feet.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70. Model Rocket Repeat Exercise 69 for a height of 100 feet.

Challenge Problems

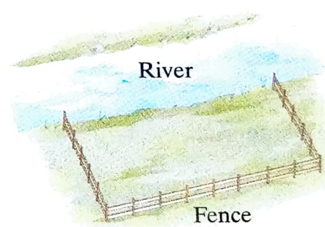
Find all the values of c that will result in each equation having **a)** *two real number solutions,* **b)** *one real number solution, and* **c)** *no real number solution.*

71. $x^2 + 8x + c = 0$

72. $2x^2 + 3x + c = 0$

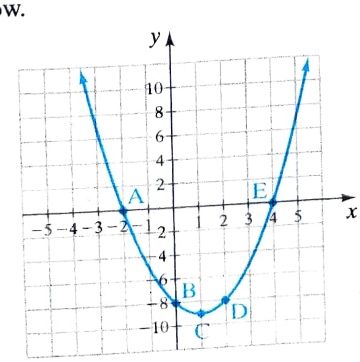
73. $-3x^2 + 6x + c = 0$

74. Fenced in Area Farmer Justina Wells wishes to form a rectangular region along a river bank by constructing fencing on three sides, as illustrated in the diagram. If she has only 400 feet of fencing and wishes to enclose an area of 15,000 square feet, find the dimensions of the rectangular region.



Group Activity

75. In Section 10.4 we will graph quadratic equations. We will learn that the graphs of quadratic equations are *parabolas*. The graph of the quadratic equation $y = x^2 - 2x - 8$ is illustrated below.



a) Each member of the group, copy the graph in your notebook.

b) Group Member 1: List the ordered pairs corresponding to points A and B. Verify that each ordered pair is a solution to the equation $y = x^2 - 2x - 8$.

c) Group Member 2: List the ordered pairs corresponding to points C and D. Verify that each ordered pair is a solution to the equation $y = x^2 - 2x - 8$.

d) Group Member 3: List the ordered pair corresponding to point E. Verify that the ordered pair is a solution to the equation $y = x^2 - 2x - 8$.

e) Individually, graph the equation $y = 2x - 3$ on the same axes that you used in part **a)**. Compare your graphs with the other members of your group.

f) The two graphs represent the system of equations

$$y = x^2 - 2x - 8$$
$$y = 2x - 3$$

As a group, estimate the points of intersection of the graphs.

g) If we set the two equations equal to each other, we obtain the following quadratic equation in only the variable x.

$$x^2 - 2x - 8 = 2x - 3$$

As a group, solve this quadratic equation. Does your answer agree with the x-coordinates of the points of intersection from part **f)**?

h) As a group, use the values of x found in part **g)** to find the values of y in $y = x^2 - 2x - 8$ and $y = 2x - 3$. Does your answer agree with the y-coordinates of the points of intersection from part **f)**?

Cumulative Review Exercises

[5.6, 10.2, 10.3] *Solve the following quadratic equations by* **a)** *factoring,* **b)** *completing the square, and* **c)** *the quadratic formula. If the equation cannot be solved by factoring, so state.*

76. $x^2 - 14x + 40 = 0$

77. $6x^2 + 11x - 35 = 0$

78. $2x^2 + 3x - 4 = 0$

79. $3x^2 = 48$

[6.4] **80.** Subtract $\dfrac{x}{2x^2 + 7x - 4} - \dfrac{2}{x^2 - x - 20}$

Mid-Chapter Test: 10.1–10.3

To find out how well you understand the chapter material to this point, take this brief test. The answers, and the section where the material was initially discussed, are given in the back of the book. Review any questions that you answered incorrectly.

Solve.

1. $x^2 = 49$

2. $a^2 = 21$

3. $16m^2 + 10 = 25$

4. $(y - 3)^2 = 4$

5. $(z + 6)^2 = 81$

6. $(b - 7)^2 = 24$

7. Numbers The product of two positive numbers is 40. Determine the numbers if the larger number is 2.5 times the smaller number.

8. Numbers When 2 times a number is added to the square of the number, the sum is 8. Find the number(s).

Solve by completing the square.

9. $x^2 + 2x - 15 = 0$

10. $x^2 + 11x + 18 = 0$

11. $p^2 - 8p = 0$

12. $h^2 + 2h - 6 = 0$

13. $x^2 - 9x + 1 = 0$

Solve by using the quadratic formula.

14. $x^2 - 3x - 40 = 0$

15. $x^2 + 13x + 42 = 0$

16. $m^2 - 5m - 2 = 0$

17. Under what conditions will a quadratic equation have

a) two real number solutions

b) one real number solution,

c) no real number solution?

Determine whether each equation has two distinct real number solutions, one real number solution, or no real number solution.

18. $3x^2 - x - 2 = 0$

19. $\dfrac{1}{2}x^2 + 4x + 11 = 0$

20. Rectangle The length of the rectangular screen on a portable DVD player is 3 inches longer than the width. Find the length and width if the screen's area is 88 square inches.