6.8 Variation

1. Set up and solve direct variation problems.
2. Set up and solve inverse variation problems.

Variation equations show how one quantity changes in relation to another quantity or quantities. In this section we will discuss two types of variation: *direct* and *inverse*. In the exercises, we will address two additional types of variation: *joint* and *combined*.

1 Set Up and Solve Direct Variation Problems

Direct variation *involves two variables that increase together or decrease together.* For example, consider a car traveling 80 miles per hour on an interstate highway. The car travels

- 80 miles in 1 hour,
- 160 miles in 2 hours,
- 240 miles in 3 hours, and so on.

As the *time* increases, the *distance* also increases.

The formula used to calculate distance traveled is

$$\text{distance} = \text{rate} \cdot \text{time}$$
$$d = rt$$

Since the rate in the example above is constant, the formula can be written

$$d = 80t$$

We say distance *varies directly* as time or that distance is *directly proportional* to time.

Understanding Algebra

Direct variation involves two variables that increase together or decrease together.

The phrases

- "y varies directly as x" and
- "y is directly proportional to x"

are both represented by the direct variation equation

$$y = kx.$$

Direct Variation

If a variable y varies directly as a variable x, then

$$y = kx$$

where k is the **constant of proportionality** or the **variation constant**.

EXAMPLE 1 **Heating Up a Hot Tub** When the heater is turned on to warm the water in a hot tub, the temperature of the water, w, increases directly with the length of time in minutes, t, the heater is on.

a) Write the variation equation.

b) If the constant of proportionality, k, is 0.8, find the increase in temperature of the water after 40 minutes.

Solution

a) We are told that the water temperature varies directly with the time. Thus we set up the direct variation equation as follows.

$$w = kt$$

b) To find the increase in water temperature, we will substitute 0.8 for k and 40 for t.

$$w = kt$$
$$w = 0.8(40) = 32$$

Thus, after 40 minutes the water temperature has increased by 32°.

Now Try Exercise 35

In many variation problems you will first have to solve for the constant of proportionality before you can solve for the variable you are asked to find. To determine the constant of proportionality, substitute the values given for the variables, and solve for k.

EXAMPLE 2 **Direct Variation Problem** s varies directly as the square of m. If $s = 125$ when $m = 5$, find s when $m = 12$.

Solution Understand and Translate We begin by setting up the variation equation. Notice that we are told that s varies directly as the square of m. The square of m is written m^2. Therefore, the equation is $s = km^2$. Since we are not given the constant of proportionality, we find it by substituting the values we are given for the variables.

$$s = km^2$$
$$125 = k(5^2) \quad \text{Substituted values.}$$
$$125 = k(25)$$
$$125 = 25k$$
$$5 = k$$

Now that we have determined k, we can answer the question by substituting 5 for k, and 12 for m.

Carry Out

$$s = km^2$$
$$s = 5(12)^2$$
$$s = 5(144)$$
$$s = 720$$

Answer Thus, when $m = 12$, $s = 720$.

Now Try Exercise 27

EXAMPLE 3 **Drug Dosage** The amount of a drug, d, given to a person is directly proportional to the person's weight, w. If an adult who weighs 75 kilograms is given 300 milligrams (mg) of the drug, determine how many milligrams of the drug are given to an adult who weighs 96 kg.

Solution Understand and Translate We are told that the amount of the drug is directly proportional to the person's weight. Thus we set up the equation

$$d = kw$$

Now we determine k by substituting the values given for d and w.

Carry Out

$$d = kw$$
$$300 = k(75)$$
$$\frac{300}{75} = k$$
$$4 = k$$

Now we proceed to find the number of milligrams of the drug to be given by substituting 4 for k and 96 for w.

$$d = kw$$
$$d = 4(96) = 384$$

Check and Answer Since we expect the amount of the drug to be greater than 300 milligrams, our answer is reasonable. A 96-kg adult should be given 384 milligrams of the drug.

Now Try Exercise 39

Understanding Algebra

Inverse variation involves two variables in which one variable increases as the other decreases and vice versa.

The phrases

- "y varies inversely as x" and
- "y is inversely proportional to x"

are both represented by the inverse variation equation

$$y = \frac{k}{x}.$$

2 Set Up and Solve Inverse Variation Problems

***Inverse variation** involves two variables in which one variable increases as the other decreases and vice versa.* For example, consider traveling 120 miles in a car. If the car is traveling

- 30 miles per hour, the trip takes 4 hours,
- 40 miles per hour, the trip takes 3 hours,
- 60 miles per hour, the trip takes 2 hours, and so on.

As the *rate* increases, the time to travel 120 miles decreases.

The formula used to calculate time, given the distance and the rate is

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

Since the distance in the example above is constant, the formula can be rewritten

$$\text{time} = \frac{120}{\text{rate}}$$

We say time *varies inversely* as rate or that time is *inversely proportional* to rate.

EXAMPLE 4 Chartering a Sailboat The cost per person for chartering a sailboat, c, is inversely proportional to the number of people chartering the boat, n. If 8 friends decide to charter the boat, the cost per person is \$60. Determine the cost per person if 15 friends decide to charter the boat.

Solution **Understand and Translate** We are told this is an example of inverse variation. Therefore, we will set up an equation to represent the inverse proportion.

$$c = \frac{k}{n}$$

Since we are not given the constant of proportionality, we determine k by substituting the values given for c and n.

$$60 = \frac{k}{8}$$
$$480 = k$$

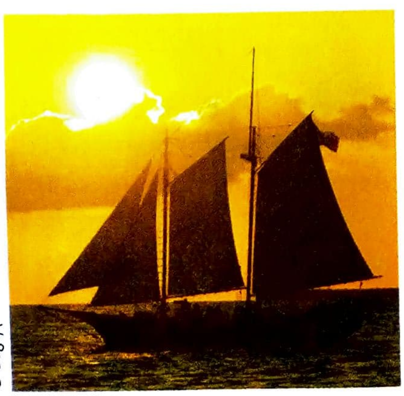
Now we can determine the answer to the question by using $k = 480$ and $n = 15$.

Carry Out

$$c = \frac{k}{n}$$
$$c = \frac{480}{15}$$
$$c = 32$$

Check and Answer The cost to each person would be \$32 if 15 friends decided to charter the sailboat.

Now Try Exercise 41



EXAMPLE 5 **Speaker Loudness** The loudness, l, of a stereo speaker, measured in decibels (dB), varies inversely as the square of the distance, d, of the listener from the speaker. Assume that for a particular speaker the loudness is 20 dB when the listener is 6 feet from the speaker.

a) Determine an equation that expresses the relationship between the loudness and the distance.

b) Using the equation obtained in part **a)**, determine the loudness when a person is 3 feet from the speaker.

Solution This problem is broken down into two parts. The first part asks us to find a general formula, while the second part asks us to use the formula.

a) **Understand and Translate** We are told that the loudness varies inversely as the *square* of the distance. Thus we write the following equation and solve for k.

Carry Out

$$l = \frac{k}{d^2}$$

$$20 = \frac{k}{6^2}$$

$$20 = \frac{k}{36}$$

$$720 = k$$

Check and Answer The constant of proportionality, k, is 720. Since for this speaker $k = 720$, the equation we are seeking is

$$l = \frac{720}{d^2}$$

b) **Understand and Translate** In part **a)** we determined the equation used to find the loudness. We substitute 3 for d in the formula and solve for l.

$$l = \frac{720}{d^2}$$

$$l = \frac{720}{3^2}$$

Carry Out

$$l = \frac{720}{9}$$

$$l = 80$$

Check and Answer Thus at 3 feet the loudness is 80 decibels. This is reasonable because at a shorter distance (3 feet versus 6 feet) the sound will be louder.

Now Try Exercise 49

Understanding Algebra

Direct variation:

As x increases so does y and as x decreases, so does y; we write $y = kx$.

Inverse variation:

As x increases, y decreases and as x decreases, y increases; we write $y = \frac{k}{x}$.

k is called the *constant of proportionality* in each case.

EXERCISE SET 6.8

MathXL® MyMathLab

Warm-Up Exercises

Fill in the blanks with the appropriate word, phrase, or symbol(s) from the following list.

directly	inversely	$y = kx$	$y = \frac{k}{x}$	uniformly	$x = ky$

1. "The variable y varies inversely as x" can be expressed as ____________.

2. "The variable y varies directly as x" can be expressed as ____________.

3. "The speed of a car and the amount of time it takes to travel a specific distance" is an example of speed varying ____________ with respect to time.

4. The formula for the surface area (A) of a cube with sides of measure s is $A = 6s^2$. Thus, A varies ____________ with respect to the square of the side.

Practice the Skills

Determine if the following are examples of direct variation or inverse variation.

5. The radius of a hose and the amount of water coming out of the hose.
6. The lens opening on a camera and the amount of light reaching the film.
7. The speed of a turtle and the length of time it takes the turtle to cross a road.
8. The age of a car, up to 8 years old, and the value of a car.
9. The temperature of water and the time it takes for an ice cube placed in the water to melt.
10. The number of people in line at the rock concert and the time it takes for all people in line to purchase tickets.

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11. The length of a roll of Scotch tape and the number of 2-inch strips that can be obtained from the roll.
12. A person's reading speed and the time it takes to read a novel.
13. The cubic-inch displacement, in liters, and the horsepower of the engine.
14. The speed of a riding lawn mower and the time it takes to cut the lawn.

In Exercises 15–22, find the quantity indicated.

15. x varies directly as z. Find x when $z = 11$ and $k = 40$.
16. x varies directly as y. Find x when $y = 9$ and $k = 6$.
17. x varies inversely as y. Find x when $y = 25$ and $k = 5$.
18. R varies inversely as W. Find R when $W = 80$ and $k = 120$.
19. C varies directly as the square of Z. Find C when $Z = 5$ and $k = 3$.
20. L varies directly as the square of R. Find L when $R = 9$ and $k = 2$.
21. y varies inversely as the square of x. Find y when $x = 10$ and $k = 250$.
22. y varies inversely as the square of w. Find y when $w = 8$ and $k = 288$.

For Exercises 23–30, find the quantity indicated.

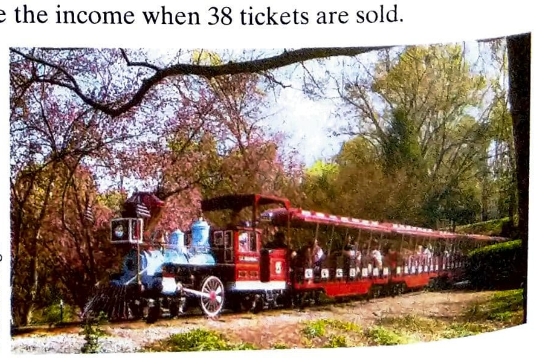
23. x varies directly as y. If $x = 9$ when $y = 27$, find x when $y = 60$.
24. Z varies directly as W. If $Z = 7$ when $W = 21$, find Z when $W = 51$.
25. C varies inversely as J. If $C = 7$ when $J = 1$, find C when $J = 2$.
26. H varies inversely as L. If $H = 15$ when $L = 60$, find H when $L = 10$.
27. y varies directly as the square of R. If $y = 4$ when $R = 4$, find y when $R = 12$.
28. A varies directly as the square of B. If $A = 245$ when $B = 7$, find A when $B = 9$.
29. L varies inversely as the square of P. If L is 320 when $P = 20$, find L when $P = 40$.
30. x varies inversely as the square of P. If $x = 10$ when $P = 6$, find x when $P = 20$.

Problem Solving

31. Assume a varies directly as b. If b is doubled, how will it affect a? Explain.
32. Assume a varies directly as b^2. If b is doubled, how will it affect a? Explain.
33. Assume y varies inversely as x. If x is doubled, how will it affect y? Explain.
34. Assume y varies inversely as a^2. If a is doubled, how will it affect y? Explain.

In Exercises 35–54, determine the quantity you are asked to find.

35. **Distance and Speed** The distance, d, a car travels is directly proportional to the speed, s, the car is traveling. Determine the distance traveled if the constant of proportionality, k, is 2 and the speed is 55 miles per hour.
36. **Swimming Pool** The time, t, it takes to fill an inground pool is directly proportional to the amount of water coming out of the hose, w. Determine the time it takes a hose to fill the pool if the constant of proportionality is 0.3 and the amount of water coming out of the hose is 200 gallons per hour.
37. **College Tuition** The amount of tuition a part-time college student is billed, A, varies directly as the number of credits, c, the student is taking. If a student is billed \$1520 for 8 credits, how much would a student be billed for taking 10 credits?
38. **Light through Water** The percent of light that filters through water, l, is inversely proportional to the depth of the water, d. Determine the percent of light that filters down to a depth of 10 feet if the constant of proportionality is 200.
39. **Kiddie Train** The income, I, for a kiddie train at an amusement park is directly proportional to the number of tickets sold, n. If the income, I, is \$33 when 22 tickets are sold, determine the income when 38 tickets are sold.



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40. **Lawn Mowing** The time it takes Sue to mow her lawn, t, is directly proportional to the area of the lawn, A. If it takes Sue 1 hour to cut an area of 2400 square feet, how long will it take her to cut an area of 1800 square feet?

41. **Roofing** The time, t, it takes to nail in shingles on a roof is inversely proportional to the number of people nailing in the shingles, n. When three people are nailing in the shingles it takes 7 hours to complete the job. How long will it take to complete the job if five people are nailing in the shingles?

42. **Baking a Turkey** The time, t, it takes to bake a turkey is inversely proportional to the oven temperature, T. If it takes 3 hours to bake a turkey at 300°F, how long will it take to bake the turkey at 250°F?

43. **Baseball Gate Receipts** The receipts r, at an International League baseball park are directly proportional to the number of people attending the game, n. If the receipts for a game are \$37,200 when 1200 people attend, determine how many people attend if the receipts for a game are \$31,000.

44. **Daily Newspaper** The time, t, it takes to print a specific number of copies of a daily newspaper is inversely proportional to the number of presses it has working, n. When 6 presses are working, the newspapers are printed in 8 hours. Determine the number of presses working if the newspapers are printed in 3 hours.

45. **Powerboats** The horsepower it takes to propel a speedboat, P, is directly proportional to the square of the velocity, v, of the boat. If it takes 900 horsepower for the boat to travel at 45 mph, what horsepower is needed to propel the speedboat at 54 mph?

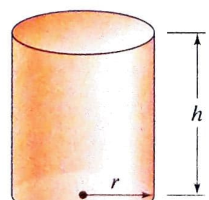
46. **Cleaning Windows** The time, t, it takes to clean all the windows in a large office building is inversely proportional to the number of teams, n, of window workers used. If 6 teams can clean all the windows in 20 days, how many teams are used if the windows are cleaned in 12 days?

47. **Area of a Circle** The area of a circle, A, is directly proportional to the square of the radius of a circle, r. If the area of a circle is about 78.5 square inches when the radius is 5 inches, determine the area when the radius is 12 inches.

48. **Falling Object** The velocity, v, of a falling object is directly proportional to the square of the time, t, it has been in free fall. An object that has been in free fall for 2 seconds has a velocity of 64 feet per second. Determine the velocity of an object that has been falling for 8 seconds.

49. **Electrical Circuit** In an electrical circuit the resistance, r, of an appliance is inversely proportional to the square of the current, c. If the resistance is 100 ohms when the current is 0.4 amps, determine the resistance if the current is 0.6 amps.

50. **Volume of a Cylinder** For a cylinder of a specific volume, the height, h, of the cylinder is inversely proportional to the square of the radius of the cylinder, r. When the radius is 6 inches, the height is 10 inches. Determine the height when the radius is 5 inches.



51. **Finding Interest** The amount of interest earned on an investment, I, varies directly as the interest rate, r. If the interest earned is \$40 when the interest rate is 4%, find the amount of interest earned when the interest rate is 5%.