3.1 Changing Application Problems into Equations

1. Translate phrases into mathematical expressions.
2. Express the relationship between two related quantities.
3. Write expressions involving multiplication.
4. Translate applications into equations.

1 Translate Phrases into Mathematical Expressions

Helpful Hint

Study Tip

It is important that you prepare for this chapter carefully. Make sure you read the book and work the examples carefully. *Attend class every day, and most of all, work all the exercises assigned to you.*

As you read through the examples in the rest of the chapter, think about how they can be expanded to other, similar problems. For example, in Example 1 **a)** we will state that the distance, d, increased by 10 miles, can be represented by $d + 10$. You can generalize this to other, similar problems. For example, a weight, w, increased by 15 pounds, can be represented as $w + 15$.

One practical advantage of knowing algebra is that you can use it to solve everyday problems by first *translating application problems into mathematical language*. One purpose of this section is to help you take an application problem, also referred to as a *word* or *verbal problem*, and write it as a mathematical equation.

Often the most difficult part of solving an application problem is translating it into an equation. Before you can translate a problem into an equation, you must understand the meaning of certain words and phrases and how they are expressed mathematically. **Table 3.1** is a list of selected words and phrases and the operations they imply. We used the variable x. However, any variable could have been used.

TABLE 3.1

Word or Phrase	Operation	Statement	Algebraic Form
Added to	Addition	8 *added to* a number	$x + 8$
More than		6 *more than* a number	$x + 6$
Increased by		A number *increased by* 3	$x + 3$
The sum of		*The sum of* a number and 4	$x + 4$
Subtracted from	Subtraction	6 *subtracted from* a number	$x - 6$
Less than		2 *less than* a number	$x - 2$
Decreased by		A number *decreased by* 5	$x - 5$
The difference between		*The difference between* a number and 9	$x - 9$
Multiplied by	Multiplication	A number *multiplied by* 6	$6x$
The product of		*The product of* 4 and a number	$4x$
Twice a number, 3 times a number, etc.		*Twice a number*	$2x$
Of, when used with a percent or fraction		20% *of* a number	$0.20x$
Divided by	Division	A number *divided by* 8	$\frac{x}{8}$
The quotient of		*The quotient of* a number and 6	$\frac{x}{6}$

Often a statement contains more than one operation. The following chart provides some examples of this.

Statement	Algebraic Form
Four more than twice a number	$\underbrace{2x}_{\text{Twice a number}} + 4$
Five less than 3 times a number	$\underbrace{3x}_{\text{Three times a number}} - 5$
Three times the sum of a number and 8	$3(\underbrace{x + 8}_{\text{The sum of a number and 8}})$
Twice the difference between a number and 4	$2(\underbrace{x - 4}_{\text{The difference between a number and 4}})$

Understanding Algebra

Subtraction is *not* commutative. For example, $6 - 9$ is not equal to $9 - 6$. *Thus, $x - 7$ is different from $7 - x$, so beware!*

Avoiding Common Errors

Subtraction is not commutative. That is, $a - b \neq b - a$. Therefore, you must be very careful when writing expressions involving subtraction. Study the following examples.

5 less than 3 times a number

CORRECT	INCORRECT
$3x - 5$	$\cancel{5 - 3x}$

5 subtracted from 3 times a number

CORRECT	INCORRECT
$3x - 5$	$\cancel{5 - 3x}$

the difference between $3x$ and 5

CORRECT	INCORRECT
$3x - 5$	$\cancel{5 - 3x}$

Often an algebraic expression can be written in several different ways.

Algebraic Expression	Statements
$2x + 3$	Three more than twice a number The sum of twice a number and 3 Twice a number, increased by 3 Three added to twice a number
$3x - 4$	Four less than 3 times a number Three times a number, decreased by 4 The difference between 3 times a number and 4 Four subtracted from 3 times a number

EXAMPLE 1 Express each statement as an algebraic expression.

a) The distance, d, increased by 10 miles

b) Six times the height, h

c) Eight less than twice the area, a

d) Four pounds more than 5 times the weight, w

Solution

a) $d + 10$ **b)** $6h$ **c)** $2a - 8$ **d)** $5w + 4$

Now Try Exercise 13

2 Express the Relationship between Two Related Quantities

When two numbers are related to each other, we will often represent one number as x and the other number as an expression containing x.

Statement	One Number	Other Number
John is 5 years older than Mary	Let x = Mary's age	Let $x + 5$ = John's age
Mike's age now and Mike's age in 8 years	Let x = Mike's age now	Let $x + 8$ = Mike's age in 8 years
The first number is 6 times the second number	Let x = second number	Let $6x$ = the first number
The first number is 12% less than the second number	Let x = second number	Let $x - 0.12x$ = the first number

Understanding Algebra

It is important to first determine what quantity we are representing with a letter. Then we should express it literally as "Let x = "

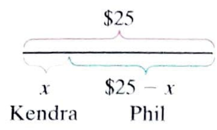

FIGURE 3.1

Understanding Algebra

In general, if T represents the total to be divided into two parts, then if one part is called x, the other part will be $T - x$. See **Figure 3.2**.

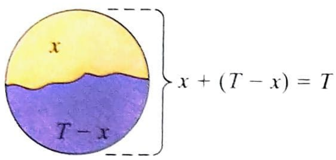

FIGURE 3.2

Now let's consider a problem in which one quantity is divided into two parts. For example, suppose \$25 is divided between Kendra and Phil.

If Kendra gets . . .	Then Phil gets . . .
\$20	\$25 − \$20 or \$5
\$15	\$25 − \$15 or \$10
\$8	\$25 − \$8 or \$17
\$2	\$25 − \$2 or \$23

In general, if we let x = the amount Kendra gets, then $25 - x$ = is the amount Phil gets. Note that the sum of x and $25 - x$ is 25. (See **Figure 3.1**.)

EXAMPLE 2 For each part, determine what to let $x =$.

a) Mary weighs 15 pounds more than Sue.

b) The length of a rectangle is 4 inches more than twice its width.

c) Joe earns \$56.20 less than Larry.

d) The theater sold 525 tickets. Some were adults' tickets and some were children's tickets.

Solution In general, when writing expressions to represent word problems, if a quantity A is expressed in terms of a quantity B, then we let x = quantity B.

a) Since Mary's weight is expressed in terms of Sue's weight, we let x = Sue's weight.

b) Since the length of a rectangle is expressed in terms of its width, we let x = width of the rectangle.

c) Since the amount Joe earns is expressed in terms of what Larry earns, we let x = amount Larry makes.

d) Here, since neither quantity of adults' tickets nor children's tickets is expressed in terms of the other's quantity, we can let x = number of adults' tickets sold or let x = number of children's tickets sold. If we let x = number of adults' tickets sold, then $525 - x$ will equal the number of children's tickets sold. If we let x = number of children's tickets sold, then $525 - x$ will equal the number of adults' tickets sold.

Now Try Exercise 37

EXAMPLE 3 For each relationship, select a variable to represent one quantity and state what that variable represents. Then express the second quantity in terms of the variable selected.

a) The Dukes scored 12 points more than the Tigers.

b) An adult robin is 4.3 times the weight of a baby robin.

c) Bill and Mary share \$75.

d) Kim has 7 more than 5 times the amount Sylvia has.

e) The length of a rectangle is 3 feet less than 4 times its width.

Solution To express the relationships, we must first decide which quantity we will let the variable represent. To give you practice with variables other than x, we will select different letters to represent the variable.

a) Since the number of points scored by the Dukes is expressed in terms of the number of points scored by the Tigers, we will select the variable t.

Let t = number of points scored by the Tigers.
Then $t + 12$ = number of points scored by the Dukes.

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b) The weight of an adult robin is given in terms of the weight of a baby robin.

$$\text{Let } w = \text{weight of a baby robin.}$$
$$\text{Then } 4.3w = \text{weight of an adult robin.}$$

c) We are not told how much of the \$75 each person receives. In this case we can let the variable represent the amount either person receives. We will let a represent the amount Bill receives.

$$\text{Let } a = \text{amount Bill receives.}$$
$$\text{Then } 75 - a = \text{amount Mary receives.}$$

d) The amount Kim has is given in terms of the amount Sylvia has.

$$\text{Let } s = \text{amount Sylvia has.}$$
$$\text{Then } 5s + 7 = \text{amount Kim has.}$$

e) The length of the rectangle is given in terms of the width of the rectangle.

$$\text{Let } w = \text{width of the rectangle.}$$
$$\text{Then } 4w - 3 = \text{length of the rectangle.}$$

Now Try Exercise 47

3 Write Expressions Involving Multiplication

Consider the statement "the cost of 3 items at \$5 each." The cost would be 3 times \$5 and we could express the cost as $3 \cdot 5$ or $3(5)$. Now consider the statement "the cost of x items at \$5 each." The cost would be x times \$5 and we could express the cost as $x \cdot 5$ or $x(5)$. It is customary to write this product as $5x$. Thus, the cost of x items at \$5 each is represented as $5x$.

Finally, consider the statement "the cost of x items at y dollars each." We write the cost of x items at y dollars as $\boldsymbol{xy}$.

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EXAMPLE 4 Write each statement as an algebraic expression.

a) The cost of purchasing x pens at \$2 each

b) A 5% commission on x dollars in sales

c) The dollar amount earned in h hours if a person earns \$6.50 per hour

d) The number of cents in q quarters

e) The number of ounces in x pounds

Solution

a) We can reason like this:

1 pen would cost	1(2) dollars	= \$2
2 pens would cost	2(2) dollars	= \$4
3 pens would cost	3(2) dollars	= \$6
⋮	⋮	⋮
x pens would cost	x(2) dollars	or $\mathbf{2x}$ dollars

Thus the cost would be $2x$ dollars.

b) A 5% commission on \$1 sales would be 0.05(1), on \$2 sales 0.05(2), on \$3 sales 0.05(3), on \$4 sales 0.05(4), and so on. Therefore, the commission on sales of x dollars would be $0.05(x)$ or $0.05x$.

Note: If you need a review on changing a percent to a decimal number, review Appendix A.

c) In one hour the person would earn 1(\$6.50). In two hours the person would earn 2(\$6.50), and in h hours the person would earn h(\$6.50) or \$6.50h.

d) We know that each quarter is worth 25 cents. Thus, one quarter is $1(25)$ cents. Two quarters is $2(25)$ cents, and so on. Therefore, q quarters is $q(25)$ cents or $25q$ cents.

e) Each pound is equal to 16 ounces. Using the same reasoning as in part **d)**, we see that x pounds is $16x$ ounces.

Now Try Exercise 71

EXAMPLE 5 **Truck Rental** Maria Mears rented a truck for 1 day. She paid a daily fee of \$88 and a mileage fee of 75 cents per mile. Write an expression that represents her total cost when she drives x miles.

Solution Maria's total cost consists of two parts, the daily fee and the mileage fee. Notice the daily fee is given in terms of dollars, and the mileage fee is given in cents. When writing an expression to represent the total cost, we want the units to be the same. Therefore, we will use a mileage fee of \$0.75 per mile, which is equal to 75 cents per mile.

Understanding Algebra

Caution! Be sure that the units are consistent throughout the problem!

Let x = number of miles driven.
Then $0.75x$ = cost of driving x miles.

$$\overbrace{\text{daily fee} + \text{mileage fee}}^{\text{total cost}}$$
$$88 \quad + \quad 0.75x$$

Thus, the expression that represents Maria's total cost is $88 + 0.75x$.

Now Try Exercise 75

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EXAMPLE 6 **Write a Sum or Difference** In a bus the number of males was 3 more than twice the number of females. Write an expression for

a) the sum of the number of males and females

b) the difference between the number of males and females

c) the difference between the number of females and males

Solution Since the number of males is expressed in terms of the number of females, we let the variable represent the number of females. We will choose x to represent the variable.

Let x = number of females.
Then $2x + 3$ = number of males.

a) The expression for the sum of the number of males and females is

$$\overbrace{(2x + 3)}^{\substack{\text{number}\\ \text{of males}}} + \overbrace{x}^{\substack{\text{number}\\ \text{of females}}}$$

b) The expression for the difference between the number of males and females is

$$\overbrace{(2x + 3)}^{\substack{\text{number}\\ \text{of males}}} - \overbrace{x}^{\substack{\text{number}\\ \text{of females}}}$$

c) The expression for the difference between the number of females and males is

$$\overbrace{x}^{\substack{\text{number}\\ \text{of females}}} - \overbrace{(2x + 3)}^{\substack{\text{number}\\ \text{of males}}}$$

Notice that parentheses are needed around the $2x + 3$ since both terms $2x$ and 3 are being subtracted.

Now Try Exercise 87

Helpful Hint

Using Parentheses When Writing Expressions

Sum: When writing the sum of two quantities, parentheses may be used to help in the understanding of the problem, but they are not necessary.

Examples	Answers
Find the sum of x and $2x - 3$.	$x + (2x - 3)$ or $x + 2x - 3$
Find the sum of $3c - 4$ and $c + 5$.	$(3c - 4) + (c + 5)$ or $3c - 4 + c + 5$

Difference: When writing the difference of two quantities, when only *a single term* is being subtracted, parentheses may be used in the understanding of the problem, but they are not necessary.

Examples	Answers
Subtract r from $3r - 2$.	$(3r - 2) - r$ or $3r - 2 - r$
Find the difference between $2s + 6$ and s.	$(2s + 6) - s$ or $2s + 6 - s$

When writing the difference of two quantities, *when two or more terms are being subtracted, parentheses **must** be placed around all the terms being subtracted,* since all the terms are being subtracted and not just the first term.

Examples	Answers
Subtract $x + 2$ from $3x$.	$3x - (x + 2)$
Subtract $3t - 4$ from $5t$.	$5t - (3t - 4)$
Subtract $r - 5$ from $2r + 3$.	$(2r + 3) - (r - 5)$ or $2r + 3 - (r - 5)$
Find the difference between 6 and $m + 3$.	$6 - (m + 3)$
Find the difference between $4n - 9$ and $2n - 3$.	$(4n - 9) - (2n - 3)$ or $4n - 9 - (2n - 3)$

Expressions Involving Percent

Example 7 involves percent. Whenever we perform a calculation involving percent, we change the percent to a decimal number or a fraction first.

When shopping we may see a "25% off" sign. We assume that this means 25% off of the *original cost*, even though this is not stated. If we let c represent the original cost, then 25% of the original cost would be represented as $0.25c$. Twenty-five percent off the original cost means the original cost, c, decreased by 25% of the original cost. Twenty five percent off the original cost would be represented as $c - 0.25c$.

25% off the original cost

$$c - 0.25c$$

Original cost   decreased by   25% of the original cost

Now let's work an example involving percent.

EXAMPLE 7 Write each statement as an algebraic expression.

a) The cost of a pair of boots, c, increased by 6%

b) The population in the town of Brooksville, p, decreased by 12%

Solution

a) The question asks for the cost increased by 6%. We assume that this means the cost increased by 6% of the original cost. Therefore, the answer is $c + 0.06c$.

b) Using the same reasoning as in part **a)**, the answer is $p - 0.12p$.

Now Try Exercise 79

Understanding Algebra

If the sales tax rate is 6%, the *tax* on an item costing x dollars is $0.06x$.

The *total amount,* which is the cost of the item plus the tax on the item, is represented as $x + 0.06x$, which can also be written as $1.06x$.

Avoiding Common Errors

In Example 7 **a)** we asked you to represent a cost, c, increased by 6%. Note, the answer is $c + 0.06c$. Often, students write the answer to this question as $c + 0.06$. It is important to realize that a percent of a quantity must always be a percent multiplied by some number or letter. Some phrases involving the word percent and the correct and incorrect interpretations follow.

PHRASE	CORRECT	INCORRECT
A $7\frac{1}{2}$% sales tax on c dollars	$0.075c$	$\cancel{0.075}$
The cost, c, increased by a $7\frac{1}{2}$% sales tax	$c + 0.075c$	$\cancel{c + 0.075}$
The cost, c, reduced by 25%	$c - 0.25c$	$\cancel{c - 0.25}$

4 Translate Applications into Equations

When writing application problems as equations, the word *is* often means *is equal to* and is represented by an equals sign. Some examples of statements written as equations follow.

Statement	Equation
Six times a number *is* 42.	$6x = 42$
Five more than twice a number *is* 4.	$2x + 5 = 4$
A number decreased by 4 *is* 3 more than twice the number.	$x - 4 = 2x + 3$
The sum of a number and the number increased by 4 *is* 60.	$x + (x + 4) = 60$
Twice the difference of a number and 3 *is* the sum of the number and 20.	$2(x - 3) = x + 20$
A number increased by 15% *is* 120.	$x + 0.15x = 120$
Six less than three times a number *is* one-fourth the number.	$3x - 6 = \frac{1}{4}x$

Now let's work some examples where we write equations.

EXAMPLE 8 Translate Words into Equations Write each problem as an equation.

a) A New York City subway car has 36 seats. The number of seats in s subway cars *is* 180.

b) The number of cents in d dimes *is* 120.

c) The cost of x gallons of gasoline at \$4.20 per gallon *is* \$35.20.

Solution

a) 1 subway car has 36 seats, 2 subway cars have 72 seats, and s subway cars have $36s$ seats. Since there are 180 seats, the equation is $36s = 180$.

b) The number of cents in d dimes is $d(10)$ or $10d$. Since the number of cents in d dimes *is* 120, the equation is $10d = 120$.

c) Using similar reasoning as in parts **a)** and **b)**, the equation is $4.20x = 35.20$.

Now Try Exercise 109

Helpful Hint

In a written expression certain other words may be used in place of *is* to represent the equals sign. Some of these are *will be, was, yields,* and *gives*. For example,

"When 4 is added to a number, the sum *will be* 20" can be expressed as $x + 4 = 20$.

"Six subtracted from a number *was* $\frac{1}{2}$ the number" can be expressed as $x - 6 = \frac{1}{2}x$.

"A rental car cost \$75 per day. The cost for renting the car for x days *was* \$150" can be expressed as $75x = 150$.

Understanding Algebra

If x is an integer, then $x + 1$ represents the *next* **consecutive integer.**

If y is an even integer, then $y + 2$ represents the *next* **consecutive even integer**.

If z is an odd integer, then $z + 2$ represents the *next* **consecutive odd integer**.

EXAMPLE 9 **Translate Words into an Equation** Write the problem as an equation.

One number is 4 less than twice the other. Their sum is 14.

Solution

$$\text{Let } x = \text{one number.}$$
$$\text{Then } 2x - 4 = \text{second number.}$$

Now we write the equation using the information given.

$$\text{first number} + \text{second number} = 14$$
$$x + (2x - 4) = 14$$

Now Try Exercise 99

In Example 10 we will use the term *consecutive even integers*.

EXAMPLE 10 **Consecutive Even Integers** Write the problem as an equation.

For two consecutive even integers, the sum of the smaller and 3 times the larger is 22.

Solution First, we express the two consecutive even integers in terms of the variable.

$$\text{Let } x = \text{smaller consecutive even integer.}$$
$$\text{Then } x + 2 = \text{larger consecutive even integer.}$$

Now we write the equation using the information given.

$$\text{smaller} + 3 \text{ times the larger} = 22$$
$$x + 3(x + 2) = 22$$

Now Try Exercise 107

EXAMPLE 11 **Translate Words into an Equation** Write the problem as an equation.

One train travels 3 miles more than twice the distance another train travels. The total distance traveled by both trains is 800 miles.

Solution First express the distance traveled by each train in terms of the variable.

$$\text{Let } x = \text{distance traveled by one train.}$$
$$\text{Then } 2x + 3 = \text{distance traveled by second train.}$$

Now write the equation using the information given.

$$\text{distance of train 1} + \text{distance of train 2} = \text{total distance}$$
$$x + (2x + 3) = 800$$

Now Try Exercise 121


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EXAMPLE 12 Translate Words into an Equation Write the problem as an equation.

Lori Soushon is 4 years older than 3 times the age of her son Ron. The difference in Lori's age and Ron's age is 26 years.

Solution Since Lori's age is given in terms of Ron's age, we will let the variable represent Ron's age.

$$\text{Let } x = \text{Ron's age.}$$
$$\text{Then } 3x + 4 = \text{Lori's age.}$$

We are told that the difference in Lori's age and Ron's age is 26 years. The word *difference* indicates subtraction. Since Lori is older than Ron, we must subtract Ron's age from Lori's age to get a positive number.

$$\text{Lori's age} - \text{Ron's age} = 26$$
$$(3x + 4) - x = 26$$

Now Try Exercise 117

Example 13 will involve percent.

EXAMPLE 13 Translate Words into an Equation. Write the problem as an equation.

The 2009 property tax for Danielle's house was 3.9% greater than her property tax in 2008. Her property tax in 2009 was $4008.

Solution In this example, we will choose to use the variable t, for tax. Since the 2009 property tax is based upon the 2008 property tax, we will let the variable represent the 2008 property tax.

$$\text{Let } t = \text{2008 property tax.}$$
$$\text{Then } t + \underbrace{0.039t}_{\text{this represents the 3.9\% increase}} = \text{2009 property tax.}$$

Since Danielle's 2009 property tax *was* $4008, the equation we write is $t + 0.039t = 4008$.

Now Try Exercise 123

Helpful Hint

It is important that you understand this section and work all your assigned homework problems. You will use the material learned in this section in the next three sections and throughout the book.

EXERCISE SET 3.1

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Warm-Up Exercises

Fill in the blanks with the appropriate word, phrase, or symbol(s) from the following list.

$2c + 7$	$2c - 7$	$c + 0.07$	$c + 7$	$c + 2$	$c + 0.07c$
$7 - c$	$c - 0.07c$	equals	$0.07c$	$c - 7$	$c + 3$

1. If c represents the cost of an item, then the 7% tax on that item is represented by ________.
2. Consider the statement "Barry is seven years older than Chuck." If c represents Chuck's age, then ________ is the expression that represents Barry's age.
3. A seven-foot board is cut into two pieces. If the length of one piece is represented by c, then ________ represents the length of the other piece.
4. If c represents an odd number, the next consecutive odd number is represented by ________.