3.2 Solving Application Problems

1. Use the problem-solving procedure.
2. Set up and solve number application problems.
3. Set up and solve application problems involving money.
4. Set up and solve applications concerning percent.

1 Use the Problem-Solving Procedure

The general problem-solving procedure given in Section 1.2 can be used to solve all types of verbal problems. Below, we present the **five-step problem-solving procedure** again so you can easily refer to it. We have included some additional information under steps 1 and 2, since in this section we are going to emphasize translating application problems into equations.

Problem-Solving Procedure for Solving Applications

1. **Understand the problem.** Identify the quantity or quantities you are being asked to find.
2. **Translate the problem into mathematical language (express the problem as an equation).**
   a) Choose a variable to represent one quantity, *and write down exactly what it represents*. Represent any other quantity to be found in terms of this variable.
   b) Using the information from step a), write an equation that represents the application.
3. **Carry out the mathematical calculations (solve the equation).**
4. **Check the answer (using the *original* application).**
5. **Answer the question asked.**

Sometimes we will combine two steps in the problem-solving procedure when it helps to clarify the explanation. We may not show the check of a problem to save space. Even if we do not show a check, you should check the problem yourself and make sure your answer is reasonable and makes sense.

2 Set Up and Solve Number Application Problems

The examples presented here involve information and data but do not contain percents.

EXAMPLE 1 An Unknown Number Two subtracted from 4 times a number is 10. Find the number.

Solution **Understand** To solve this problem, we need to express the statement given as an equation. We are asked to find the unknown number.

Translate Let x = the unknown number. Now write the equation.

$$\underbrace{4x - 2}_{\text{2 subtracted from 4 times a number}} \quad \overset{\text{is}}{=} \quad 10$$

Carry Out

$$4x = 12$$
$$x = 3$$

Check Substitute 3 for the number in the original problem, two subtracted from 4 times a number is 10.

$$4(3) - 2 \stackrel{?}{=} 10$$
$$10 = 10 \quad \text{True}$$

Answer Since the solution checks, the unknown number is 3.

Now Try Exercise 7

EXAMPLE 2 **Number Problem** The sum of two numbers is 26. Find the two numbers if the larger number is 2 less than three times the smaller number.

Solution **Understand** We are given that "the larger number is 2 less than three times the smaller number." Notice that the larger number is expressed in terms of the smaller number. Therefore, we will let the variable represent the smaller number.

Translate

Let x = smaller number.

Then $3x - 2$ = larger number.

The sum of the two numbers is 26. Therefore, we write the equation

$$\text{smaller number} + \text{larger number} = 26$$
$$x + (3x - 2) = 26$$

Carry Out Now we solve the equation.

$$\begin{aligned} 4x - 2 &= 26 \\ 4x &= 28 \\ x &= 7 \end{aligned}$$

The smaller number is 7. Now we find the larger number.

$$\begin{aligned} \text{larger number} &= 3x - 2 \\ &= 3(7) - 2 \quad \text{Substitute 7 for } x. \\ &= 19 \end{aligned}$$

The larger number is 19.

Check The sum of the two numbers is 26.

$$\begin{aligned} 7 + 19 &\stackrel{?}{=} 26 \\ 26 &= 26 \quad \text{True} \end{aligned}$$

Answer The two numbers are 7 and 19.

Now Try Exercise 13

Helpful Hint

When reading a word problem, ask yourself, "How many answers are required?" In Example 2, the question asked for the two numbers. The answer is 7 and 19. It is important that you read the question and identify what you are being asked to find. If the question had asked "Find the *smaller* of the two numbers if the larger number is 2 less than three times the smaller number," then the answer would have been only the 7. If the question had asked to find the *larger* of the two numbers, then the answer would have been only 19. *Make sure you answer the question asked in the problem.*

EXAMPLE 3 **2008 Summer Olympics** In the 2008 Olympics in Beijing, China, the United States won the most medals and China won the second greatest number of medals. The United States won 90 less than twice the number of medals won by China. If the difference between the number of medals won by the United States and China was 10, determine the number of medals won by the United States.

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Solution **Understand** The word *difference* in the problem indicates that this problem will involve subtraction. We are asked to find the number of medals won by the United States. Since the number of medals won by the United States is given in terms of the number of medals won by China, we will let the variable represent medals won by China. We will use the variable c.

Translate

Let c = medals won by China.

Then $2c - 90$ = medals won by the United States.

Since we are dealing with positive amounts, we must subtract the smaller quantity from the larger. Since the difference in medals between the United States and China is 10, we write the following equation.

$$\underbrace{\text{number of medals won by U.S.}}_{2c - 90} - \underbrace{\text{number of medals won by China}}_{c} = 10$$

$$2c - 90 - c = 10$$

Carry Out

$$c - 90 = 10$$

$$c = 100$$

Check and Answer Remember c represents the number of medals won by China. We are asked to find the number of medals won by the United States, which we have represented as $2c - 90$. Now, we substitute the known value of 100 for c in the expression $2c - 90$: $2(100) - 90 = 110$. *The answer is that the United States won 110 medals.* Notice that the difference in the number of medals won by the United States and China is $110 - 100 = 10$, so the answer checks.

Now Try Exercise 27

EXAMPLE 4 Bicycles The Chain Wheel Drive Bicycle Company presently manufactures 800 bicycles a month. Each month after this month the company plans to increase production by 150 bicycles a month until its monthly production reaches 1700 bicycles. How long will it take the company to reach its production goal?

Solution Understand We are asked to find the *number of months* that it will take for the company's production to reach 1700 bicycles a month. Next month its production will increase by 150 bicycles. In two months, its production will increase by 2(150) over the present month's production. In n months, its production will increase by $n(150)$ or $150n$. We will use this information when we write the equation to solve the problem.

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Translate

$$\text{Let } n = \text{number of months.}$$

$$\text{Then } 150n = \text{increase in production over } n \text{ months.}$$

$$(\text{present production}) + \begin{pmatrix}\text{increased production} \\ \text{over } n \text{ months}\end{pmatrix} = \text{future production}$$

$$800 + 150n = 1700$$

Carry Out

$$150n = 900$$

$$n = \frac{900}{150}$$

$$n = 6 \text{ months}$$

Check and Answer As a check, let's list the number of bicycles produced this month and for the next 6 months.

Presently	Next month	Month 2	Month 3	Month 4	Month 5	Month 6
↓	↓	↓	↓	↓	↓	↓
800	950	1100	1250	1400	1550	1700

Thus, in 6 months the company will produce 1700 bicycles per month.

Now Try Exercise 23

3 Set Up and Solve Application Problems Involving Money

When setting up an equation that involves money, you must make sure that all the monetary units entered into the equation are the same, either all dollars or all cents. When pieces of information are given in both dollars and cents, we generally convert the amount given in cents to an equivalent amount of dollars. For example, when renting a truck the cost may be \$50 a day plus 90 cents a mile. When writing the equation,

we would write the 90 cents a mile as \$0.90 a mile. The cost of traveling x miles at 90 cents a mile would be written $\$0.90x$.

EXAMPLE 5 Grub Problem Part of Kim Martello's lawn was destroyed by grubs. She decided to purchase new sod to lay down. The cost of the sod is 45 cents per square foot plus a delivery charge of \$59. If the total cost of delivery plus the sod was \$284, how many square feet of sod was delivered?

Solution **Understand** The total cost consists of two parts, a cost of 45 cents per square foot of sod, plus a delivery charge of \$59. We need to determine the number of square feet of sod that will result in a total cost of \$284.

Translate

Let x = number of square feet of sod.

Then $0.45x$ = cost of x square feet of sod.

sod cost + delivery cost = total cost

Carry Out

$$0.45x + 59 = 284 \quad \text{Subtract 59 from both sides.}$$
$$0.45x = 225$$
$$\frac{0.45x}{0.45} = \frac{225}{0.45}$$
$$x = 500$$

Check The cost of 500 square feet of sod at 45 cents a square foot is $500(0.45) = \$225$. Adding the \$225 to the delivery cost of \$59 gives \$284, so the answer checks.

Answer Five hundred square feet of sod was delivered.

Now Try Exercise 35

EXAMPLE 6 Photo Printer Elsie Newman is going to purchase a photo printer to print pictures from her digital camera. She is considering a Hewlett-Packard (HP) printer and a Lexmark printer. The HP printer costs \$419 and the cost for the ink and paper is 14 cents per photo printed. The Lexmark printer costs \$299 and the cost for the ink and paper is 18 cents per photo. How many photos would need to be printed for the total cost of the printers, ink, and paper to be the same?

Solution **Understand** The HP printer has a greater initial cost (\$419 versus \$299); however, its cost per photo printed is less (14 cents versus 18 cents). We are asked to find the number of photos printed so that the total cost of the two printers will be the same.

Translate

Let n = number of photos.

Then $0.14n$ = cost for printing n photos with the HP printer

and $0.18n$ = cost for printing n photos with the Lexmark printer.

total cost of HP printer = total cost of Lexmark printer

$$\begin{pmatrix}\text{initial}\\ \text{cost}\end{pmatrix} + \begin{pmatrix}\text{cost}\\ \text{for } n \text{ photos}\end{pmatrix} = \begin{pmatrix}\text{initial}\\ \text{cost}\end{pmatrix} + \begin{pmatrix}\text{cost}\\ \text{for } n \text{ photos}\end{pmatrix}$$

$$419 + 0.14n = 299 + 0.18n$$

Carry Out

$$120 + 0.14n = 0.18n \quad \text{299 was subtracted from both sides.}$$
$$120 = 0.04n \quad \text{0.14}n \text{ was subtracted from both sides.}$$
$$\frac{120}{0.04} = \frac{0.04n}{0.04}$$
$$3000 = n$$

Check and Answer The total cost would be the same when 3000 photos were printed. We will leave the check of this answer for you.

Now Try Exercise 39

Understanding Algebra

Recall, the word *percent* means *per hundred*. So, 30% of 80 is $0.30 \times 80 = 24$.

In application problems involving percent, we are always taking percents *of* quantities. Remember, "of" means multiply.

4 Set Up and Solve Applications Concerning Percent

Now we'll look at some application problems that involve percent. Remember that a percent is always a percent of something. Thus if the cost of an item, c, is increased by 8%, we would represent the new cost as $c + 0.08c$, and not $c + 0.08$. See the Avoiding Common Errors box on page 178.

EXAMPLE 7 Water Bike Rental At a beachfront hotel, the cost for a water bike rental is \$30 per half hour, which includes a $7\frac{1}{2}\%$ sales tax. Find the cost of the rental before tax.

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Solution **Understand** We are asked to find the cost of the water bike rental before tax. The cost of the rental before tax plus the tax on the water bike must equal \$30.

Translate

Let x = cost of the rental before tax.

Then $0.075x$ = tax on the rental.

(cost of the water bike rental before tax) + (tax on the rental) = 30

Carry Out

$$x + 0.075x = 30$$
$$1.075x = 30$$
$$x = \frac{30}{1.075}$$
$$x \approx 27.91$$

Check and Answer A check will show that if the cost of the rental is \$27.91, the cost of the rental including a $7\frac{1}{2}\%$ tax is about \$30.

Now Try Exercise 47

EXAMPLE 8 Caloric Intake If the caloric intake for men in the United States increased 10% from 1971 to 2009 and reached 2695 calories in 2009, determine the average caloric intake for men in 1971.

Solution **Understand** We represent the 2009 caloric intake in terms of the unknown 1971 caloric intake. We use c to represent the 1971 caloric intake and the 2009 caloric intake can be expressed in terms of c.

Translate

Let c = the 1971 caloric intake.

Then $c + 0.10c$ = the 2009 caloric intake.

Since the 2009 caloric intake is 2695 calories, we set up the following equation.

2009 caloric intake is 2695

$$\underbrace{c + 0.10c}_{\text{2009 caloric intake}} = 2695$$

Carry Out

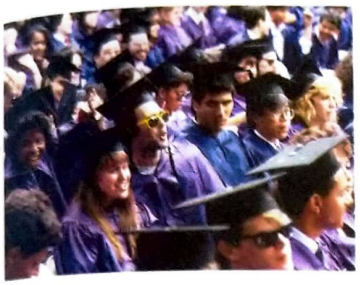
$$1.10c = 2695$$
$$c = \frac{2695}{1.10}$$
$$c = 2450$$

Check and Answer Since c represents the 1971 caloric intake, and it is less than the 2009 caloric intake, our answer is reasonable. The 1971 caloric intake for men was 2450 calories.

Now Try Exercise 49

EXAMPLE 9 Salary Plans Jeanne Pirie has accepted a position selling medical supplies and equipment. During her first year, she is given a choice of salary plans. Plan 1 is a $450 weekly base salary plus a 3% commission of weekly sales. Plan 2 is a straight 10% commission of weekly sales. What weekly amount of sales, in dollars, would result in Jeanne receiving the same salary from both plans?

Solution **Understand** We are asked to find the *amount of sales*, in dollars, that will result in the same total salary from both plans. To solve this problem, we write expressions to represent the salary from each of the plans and set the salaries equal to one another.



Translate

Let x = amount of sales in dollars.

Then $0.03x$ = commission from plan 1 sales

and $0.10x$ = commission from plan 2 sales.

salary from plan 1 = salary from plan 2

base salary + 3% commission = 10% commission

$$450 + 0.03x = 0.10x$$

Carry Out

$$450 = 0.07x$$

or $$0.07x = 450$$

$$\frac{0.07x}{0.07} = \frac{450}{0.07}$$

$$x \approx 6428.57$$

Check We will leave it up to you to show that sales of $6428.57 result in Jeanne receiving the same weekly salary from both plans.

Answer Jeanne's weekly salary will be the same from both plans if she sells $6428.57 worth of medical supplies and equipment.

Now Try Exercise 59

Helpful Hint

Here are some suggestions if you find you are having some difficulty with application problems.

1. Instructor—Make an appointment to see your instructor. Make sure you have read the material in the book and attempted all the homework problems. Go with specific questions for your instructor.
2. Tutoring—If your college learning center offers free tutoring, you may wish to take advantage of tutoring.
3. Study Group—Form a study group with classmates. Exchange phone numbers and e-mail addresses. You may be able to help one another.
4. Student's Solutions Manual—If you get stuck on an exercise you may want to use the Student's Solutions Manual to help you understand a problem. Do not use the Solutions Manual in place of working the exercises. In general, the Solutions Manual should be used only to check your work.
5. MyMathLab—MyMathLab provides exercises correlated to the text. In addition, online tools such as video lectures, animations, and a multimedia textbook are available to help you understand the material.
6. Math XL®—MathXL is a powerful online homework, tutorial, and assessment system correlated specifically to this text. You can take chapter tests in MathXL and receive a personalized study plan based on your test results. The study plan links directly to tutorial exercises for the objectives you need to study or retest.
7. Pearson Tutor Center—Once the program has been initiated by your instructor, you can get individual tutoring by phone, fax, or e-mail.

It is important that you keep trying! Remember, the more you practice, the better you will become at solving application problems.