

## 3.3 Geometric Problems

1 Solve geometric problems.

### 1 Solve Geometric Problems

This section serves two purposes. One is to reinforce the geometric formulas introduced in Section 2.6. The second is to reinforce procedures for setting up and solving verbal problems discussed in Sections 3.1 and 3.2.

**EXAMPLE 1 Sandbox** Christine O'Connor is planning to build a sandbox for her daughter. She has 30 feet of lumber with which to build the perimeter. What should be the dimensions of the rectangular sandbox if the length is to be 3 feet longer than the width (**Fig. 3.3**)?

**Solution Understand** We are asked to find the dimensions of the sandbox that Christine plans to build. The perimeter of the sandbox will be 30 feet. Since the length is given in terms of the width, we will let  $w$  represent the width. Then we can express the length in terms of  $w$ . To solve this problem, we use the formula for the perimeter of a rectangle,  $P = 2l + 2w$ , where  $P = 30$  feet.

**Translate**

Let  $w$  = width of the sandbox.

Then  $w + 3$  = length of the sandbox.

$$P = 2l + 2w$$

$$30 = 2(w + 3) + 2w$$

$$30 = 2w + 6 + 2w$$

$$30 = 4w + 6$$

$$24 = 4w$$

$$6 = w$$

**Carry Out**

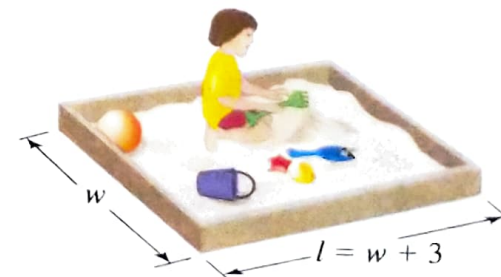


FIGURE 3.3

### Understanding Algebra

Recall some useful geometry formulas:

Rectangle Area:  $A = l \cdot w$   
Perimeter:  $P = 2l + 2w$

Triangle Area:  $A = \frac{1}{2}b \cdot h$

Sum of the measures of the interior angles =  $180^\circ$

Quadrilateral Sum of the measures of the interior angles =  $360^\circ$

Circle Area =  $\pi r^2$   
Circumference =  $2\pi r$

## Understanding Algebra

*Additional geometry facts:*

Equilateral triangle: has 3 equal sides and 3 equal angles (all  $60^\circ$ )

Isosceles triangles: has 2 equal sides

Congruent triangles: corresponding sides and corresponding angles are equal

Similar triangles: corresponding sides are in proportion and corresponding angles are equal

Polygon with  $n$  sides: sum of the measures of the interior angles is  $(n - 2)180^\circ$

Complementary angles: angles whose measures add to  $90^\circ$

Supplementary angles: angles whose measures add to  $180^\circ$

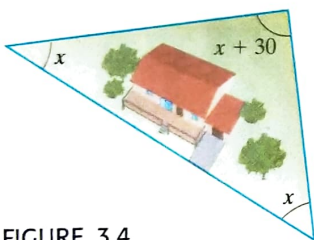


FIGURE 3.4

The width is 6 feet. Since the length is 3 feet longer than the width, the length is  $6 + 3 = 9$  feet.

**Check** We will check the solution by substituting the appropriate values in the perimeter formula.

$$P = 2l + 2w$$

$$30 \stackrel{?}{=} 2(9) + 2(6)$$

$$30 = 30 \quad \text{True}$$

**Answer** The width of the sandbox will be 6 feet and the length will be 9 feet.

Now Try Exercise 23

A triangle that contains two sides of equal length is called an **isosceles triangle**. In isosceles triangles, the angles opposite the two sides of equal length have equal measures.

**EXAMPLE 2 Corner Lot** Mr. and Mrs. Harmon Katz have a corner lot that is in the shape of an isosceles triangle. Two angles of their triangular lot are the same and the third angle is  $30^\circ$  greater than the other two. Find the measure of all three angles (see **Fig. 3.4**).

**Solution Understand** To solve this problem, you must know that the sum of the angles of any triangle measures  $180^\circ$ . We are asked to find the measure of each of the three angles, where the two smaller angles have the same measure. We will let  $x$  represent the measure of the smaller angles, and then we will express the larger angle in terms of  $x$ .

**Translate** Let  $x$  = the measure of each smaller angle.

Then  $x + 30$  = the measure of the larger angle.

$$\text{sum of the 3 angles} = 180$$

**Carry Out**

$$x + x + (x + 30) = 180$$

$$3x + 30 = 180$$

$$3x = 150$$

$$x = \frac{150}{3} = 50$$

The two smaller angles are each  $50^\circ$ . The larger angle is  $x + 30^\circ$  or  $50^\circ + 30^\circ = 80^\circ$ .

**Check and Answer** Since  $50^\circ + 50^\circ + 80^\circ = 180^\circ$ , the answer checks. The two smaller angles are each  $50^\circ$  and the larger angle is  $80^\circ$ .

Now Try Exercise 11

Recall from Section 2.6 that a quadrilateral is a four-sided figure. Quadrilaterals include squares, rectangles, parallelograms, and trapezoids. The sum of the measures of the angles of any quadrilateral is  $360^\circ$ .

**EXAMPLE 3 Water Trough** Sarah Fuqua owns horses and uses a water trough whose ends are trapezoids. The measure of the two bottom angles of the trapezoid are the same, and the measure of the two top angles are the same. The bottom angles measure  $15^\circ$  less than twice the measure of the top angles. Find the measure of each angle.

**Solution Understand** To help visualize the problem, we draw a picture of the trapezoid, as in **Figure 3.5**. We use the fact that the sum of the measures of the four angles of a quadrilateral is  $360^\circ$ .

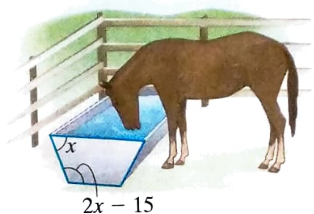


FIGURE 3.5

**Translate**Let  $x$  = the measure of each of the two smaller angles.Then  $2x - 15$  = the measure of each of the two larger angles.

$$\left( \begin{array}{l} \text{measure of the} \\ \text{two smaller angles} \end{array} \right) + \left( \begin{array}{l} \text{measure of the} \\ \text{two larger angles} \end{array} \right) = 360$$

$$x + x + (2x - 15) + (2x - 15) = 360$$

**Carry Out**

$$x + x + 2x - 15 + 2x - 15 = 360$$

$$6x - 30 = 360$$

$$6x = 390$$

$$x = 65$$

Each smaller angle is  $65^\circ$ . Each larger angle is  $2x - 15 = 2(65) - 15 = 115^\circ$ .**Check and Answer** Since  $65^\circ + 65^\circ + 115^\circ + 115^\circ = 360^\circ$ , the answer checks. Each smaller angle is  $65^\circ$  and each larger angle is  $115^\circ$ .**Now Try Exercise 27**

**EXAMPLE 4 Fenced-In Area** Ronald Yates recently started an ostrich farm. He is separating the ostriches by fencing in three equal rectangular areas, as shown in **Figure 3.6**. The length of the fenced-in area,  $l$ , is to be 30 feet greater than the width and the total amount of fencing available is 660 feet. Find the length and width of the fenced-in area.

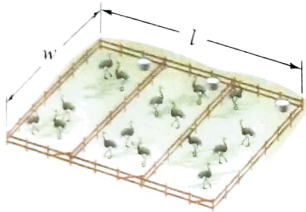


FIGURE 3.6

**Solution Understand** The fencing consists of four pieces of fence of length  $w$ , and two pieces of fence of length  $l$ .**Translate**Let  $w$  = width of fenced-in area.Then  $w + 30$  = length of fenced-in area.

$$\left( \begin{array}{l} 4 \text{ pieces of fence} \\ \text{of length } w \end{array} \right) + \left( \begin{array}{l} 2 \text{ pieces of fence} \\ \text{of length } w + 30 \end{array} \right) = 660$$

**Carry Out**

$$4w + 2(w + 30) = 660$$

$$4w + 2w + 60 = 660$$

$$6w + 60 = 660$$

$$6w = 600$$

$$w = 100$$

Since the width is 100 feet, the length is  $w + 30$  or  $100 + 30$  or 130 feet.**Check and Answer** Since  $4(100) + 2(130) = 660$ , the answer checks. The width of the fenced-in area is 100 feet and the length is 130 feet.**Now Try Exercise 37****EXERCISE SET 3.3****Warm-Up Exercises**

Fill in the blanks with the appropriate word, phrase, or symbol(s) from the following list.

$360^\circ$                        $180^\circ$                        $90^\circ$                        $2l + 12$                       12                      equilateral  
 10                      quadrilateral                       $l \cdot w$                        $45^\circ$                       isosceles

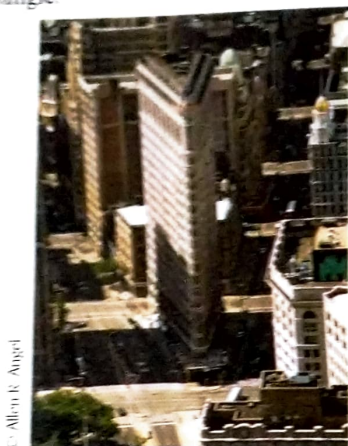
- In a rectangle, the length is 1 inch more than twice the width and the perimeter is 62 inches. Then the width is \_\_\_\_\_ inches.
- The area of a rectangle is 120 square inches. If the length is 10 inches, then the width is \_\_\_\_\_ inches.
- The sum of the measures of the interior angles of a quadrilateral is \_\_\_\_\_.
- The sum of the measures of the angles of a triangle is \_\_\_\_\_.
- A triangle with two equal sides is called \_\_\_\_\_.
- A triangle with three equal sides is called \_\_\_\_\_.
- The area of a rectangle of length  $l$  and width  $w$  is \_\_\_\_\_.
- A four-sided figure is called a \_\_\_\_\_.

9. If one angle of an isosceles triangle is  $90^\circ$ , then each other angle must measure \_\_\_\_\_.

### Practice the Skills Problem Solving

Solve the following geometric problems.\*

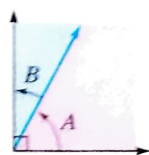
11. **Isosceles Triangle** In an isosceles triangle, one angle is  $42^\circ$  greater than the other two equal angles. Find the measure of all three angles. See Example 2.
12. **Triangular Building** This building in New York City, referred to as the Flatiron Building, has a perimeter in the shape of an isosceles triangle. If the shortest side of the triangle is 50 feet shorter than the two longer sides, and the perimeter is 196 feet, determine the length of the three sides of the triangle.



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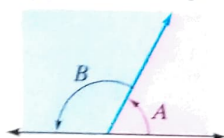
13. **A Special Triangle** An **equilateral triangle** is a triangle that has three sides of the same length. The perimeter of an equilateral triangle is 34.5 inches. Find the length of each side.
14. **Equilateral Triangle** The perimeter of an equilateral triangle is 48.6 centimeters. Find the length of each side. See Exercise 13.
15. **Complementary Angles** Two angles are **complementary angles** if the sum of their measures is  $90^\circ$ . Angle  $A$  and angle  $B$  are complementary angles, and angle  $A$  is  $21^\circ$  more than twice angle  $B$ . Find the measures of angle  $A$  and angle  $B$ .

Complementary Angles



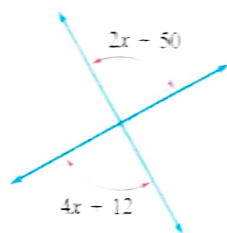
16. **Complementary Angles** Angles  $A$  and  $B$  are complementary angles, and angle  $B$  is  $14^\circ$  less than angle  $A$ . Find the measures of angle  $A$  and angle  $B$ . See Exercise 15.
17. **Supplementary Angles** Two angles are **supplementary angles** if the sum of their measures is  $180^\circ$ . Angle  $A$  and angle  $B$  are supplementary angles, and angle  $B$  is  $8^\circ$  less than three times angle  $A$ . Find the measures of angle  $A$  and angle  $B$ .

Supplementary Angles

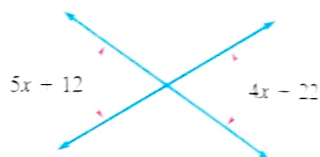


10. The perimeter of a rectangle of length  $l$  and width  $w$  is \_\_\_\_\_.

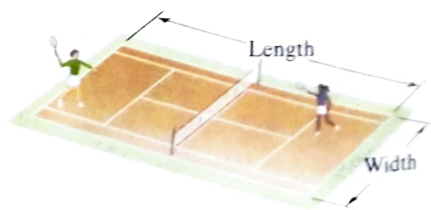
18. **Supplementary Angles** Angles  $A$  and  $B$  are supplementary angles and angle  $A$  is  $2^\circ$  more than 4 times angle  $B$ . Find the measures of angle  $A$  and angle  $B$ . See Example 17.
19. **Vertical Angles** When two lines intersect, the opposite angles are called **vertical angles**. Vertical angles have equal measures. Determine the measures of the vertical angles indicated in the following figure.



20. **Vertical Angles** A pair of vertical angles is indicated in the following figure. Determine the measure of the vertical angles indicated. See Exercise 19.



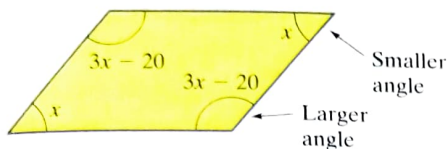
21. **Unknown Angles** One angle of a triangle is  $10^\circ$  greater than the smallest angle, and the third angle is  $30^\circ$  less than twice the smallest angle. Find the measures of the three angles.
22. **Unknown Angles** One angle of a triangle is  $20^\circ$  larger than the smallest angle, and the third angle is 6 times as large as the smallest angle. Find the measures of the three angles.
23. **Dimensions of Rectangle** The length of a rectangle is 6 feet more than its width. What are the dimensions of the rectangle if the perimeter is 44 feet?
24. **Dimensions of Rectangle** The perimeter of a rectangle is 120 feet. Find the length and width of the rectangle if the length is twice the width.
25. **Tennis Court** The length of a regulation tennis court is 7 feet greater than twice its width. The perimeter of the court is 228 feet. Find the length and width of the court.



\*See Appendix C for more material on geometry.

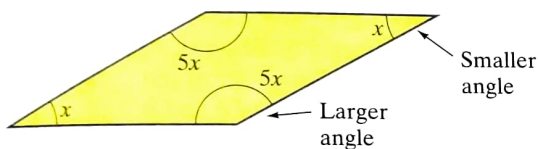
26. **Patio** Rikki Blair is building a rectangular patio. The perimeter of the patio is to be 96 feet. Determine the dimensions of the patio if the length is to be 6 feet less than twice the width.

27. **Parallelogram** In a parallelogram the opposite angles have the same measures. Each of the two larger angles in a parallelogram is  $20^\circ$  less than 3 times the smaller angles. Find the measure of each angle.



28. **Parallelogram** The two smaller angles of a parallelogram have equal measures, and the two larger angles each measure  $27^\circ$  less than twice each smaller angle. Find the measure of each angle.

29. **Rhombus** A rhombus is a parallelogram with four equal sides. Each of the two larger angles of a rhombus is 5 times as large as the two smaller angles. Find the measure of each of the four angles.

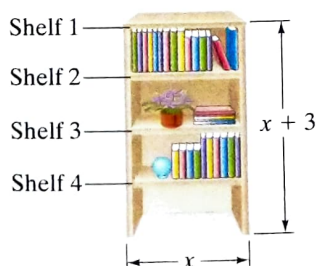


30. **Rhombus** Each of the two larger angles of a rhombus are  $20^\circ$  less than four times the two smaller angles. Find the measure of each of the four angles.

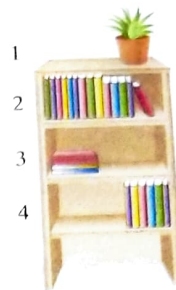
31. **Quadrilateral** The measure of one angle of a quadrilateral is  $10^\circ$  greater than the smallest angle; the third angle is  $14^\circ$  greater than twice the smallest angle; and the fourth angle is  $21^\circ$  greater than the smallest angle. Find the measures of the four angles of the quadrilateral.

32. **Quadrilateral** The measure of one angle of a quadrilateral is twice the smallest angle; the third angle is  $20^\circ$  greater than the smallest angle; and the fourth angle is  $20^\circ$  less than twice the smallest angle. Find the measures of the four angles of the quadrilateral.

33. **Building a Bookcase** A bookcase is to have four shelves, including the top, as shown. The height of the bookcase is to be 3 feet more than the width. Find the width and height of the bookcase if only 30 feet of lumber is available.

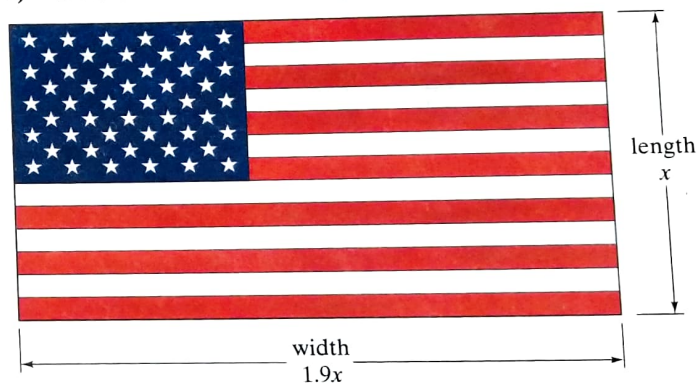


34. **Bookcase** A bookcase is to have four shelves as shown. The height of the bookcase is to be 2 feet more than the width, and only 20 feet of lumber is available. What should be the width and height of the bookcase?



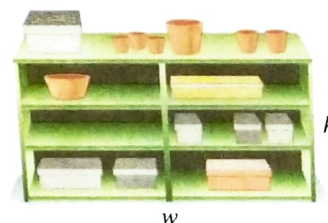
35. **American Flag** The dimensions of the American flag appear in the figure below. The perimeter of this particular flag is 580 inches.

- Determine its length and width.
- What is the width of each of the stripes?
- The (left-to-right) width of the blue rectangle is always 76% of the vertical height of the flag. How many inches is that blue width?
- The vertical height of the blue rectangle is always 53.85% of the vertical height of the flag. How many inches is the blue height?
- What is the area of this flag?



Source: [www.usflag.org](http://www.usflag.org)

36. **Storage Shelves** Carlotta Perez plans to build storage shelves as shown. She has only 45 feet of lumber for the entire unit and wishes the width to be 3 times the height. Find the width and height of the unit.



37. **Fenced-In Area** A rectangular area is to be fenced in along a straight river bank as illustrated. The length of the fenced-in area is to be 5 feet greater than the width, and the total amount of fencing to be used is 71 feet. Find the width and length of the fenced-in area.

