3.4 Motion, Money, and Mixture Problems

1. Solve motion problems involving two rates.
2. Solve money problems.
3. Solve mixture problems.

We now discuss three additional types of applications: motion, money, and mixture problems. These problems are grouped in the same section because, as you will learn shortly, you use the same general multiplication procedure to solve them. We begin by discussing motion problems.

1 Solve Motion Problems Involving Two Rates

A **motion problem** is one in which an object is moving at a specific rate for a specific period of time. Examples of motion problems include a car traveling at a constant speed or a person walking at a constant speed or a boat being rowed at a constant speed. In this section, we will discuss motion problems that involve *two rates*, such as two trains traveling at different speeds. We will use the distance formula, distance = rate × time, and construct tables like the following one to organize the information. The formula at the top of the table shows how the distance in the last column is calculated.

Understanding Algebra

Motion problems involve *rates*. Units of rates include miles per hour (mph), feet per second (ft/s) and meters per second (m/s).

The formula we use in motion problems is:

distance = rate × time

or $d = r \cdot t$.

	Rate	×	Time	=	Distance
Item	Rate		Time		Distance
Item 1					distance 1
Item 2					distance 2

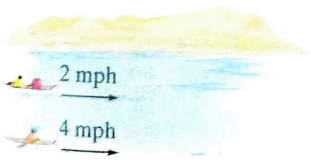
Examples 1 and 2 illustrate the procedure used.

EXAMPLE 1 Camping Trip Maryanne and Paul Justinger and their son Danny are on a canoe trip on the Erie Canal. Danny is in one canoe and Paul and Maryanne are in a second canoe. Both canoes start at the same time from the same point and travel in the same direction. The parents paddle their canoe at 2 miles per hour and their son paddles his canoe at 4 miles per hour. In how many hours will the two canoes be 5 miles apart?

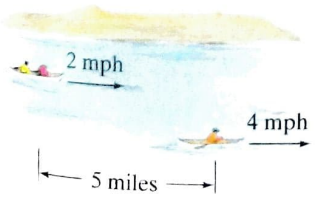
Solution Understand and Translate We are asked to find the time it takes for the canoes to become separated by 5 miles. We construct a table to aid us in setting up the problem.

Let t = time when canoes are 5 miles apart.

Draw a sketch to help visualize the problem (**Fig. 3.7**). When the two canoes are 5 miles apart, each has traveled for the same number of hours, t.



(a) Beginning of trip



(b) After t hours

FIGURE 3.7

Canoe	Rate	Time	Distance
Parents	2	t	$2t$
Son	4	t	$4t$

Since the canoes are traveling in the same direction, the distance between them is found by subtracting the distance traveled by the slower canoe from the distance traveled by the faster canoe.

$$\begin{pmatrix}\text{distance traveled}\\ \text{by faster canoe}\end{pmatrix} - \begin{pmatrix}\text{distance traveled}\\ \text{by slower canoe}\end{pmatrix} = 5 \text{ miles}$$

Carry Out

$$4t - 2t = 5$$
$$2t = 5$$
$$t = 2.5$$

Answer After 2.5 hours the two canoes will be 5 miles apart.

Now Try Exercise 7

EXAMPLE 2 Paving Roads Two highway paving crews are 20 miles apart working toward each other. One crew paves 0.4 mile of road per day more than the other crew, and the two crews meet after 10 days. Find the rate at which each crew paves the road.

Solution Understand and Translate We are asked to find the two rates. We are told that both crews work for 10 days.

Let r = rate of slower crew.

Then $r + 0.4$ = rate of faster crew.

We make a sketch (**Fig. 3.8**) and set up a table of values.

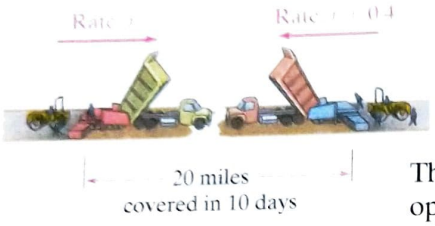


FIGURE 3.8

Crew	Rate	Time	Distance
Slower	r	10	$10r$
Faster	$r + 0.4$	10	$10(r + 0.4)$

The total distance covered by both crews is 20 miles. Since the crews are moving in opposite directions, the distance between them is found by adding the two distances.

$$\begin{pmatrix}\text{distance covered}\\ \text{by slower crew}\end{pmatrix} + \begin{pmatrix}\text{distance covered}\\ \text{by faster crew}\end{pmatrix} = 20 \text{ miles}$$

Carry Out

$$\begin{aligned} 10r \quad + \quad 10(r + 0.4) &= 20 \\ 10r + 10r + 4 &= 20 \\ 20r + 4 &= 20 \\ 20r &= 16 \\ \frac{20r}{20} &= \frac{16}{20} \\ r &= 0.8 \end{aligned}$$

Answer The slower crew paves 0.8 mile of road per day and the faster crew paves $r + 0.4$ or $0.8 + 0.4 = 1.2$ miles of road per day.

Now Try Exercise 23

Understanding Algebra

Notice that in Example 2, the rate, r, is measured in *miles* (of road) *per day*.

Helpful Hint

When working with two different moving items, if the items are moving in the same direction, the solution will involve subtracting the smaller distance from the larger distance as in Example 1. If the items are moving in opposite directions, the solution will involve adding the distances together as in Example 2.

2 Solve Money Problems

One type of money problem involves simple interest. When working with simple interest problems involving two amounts, we can use a table like the one below.

	Principal	×	Rate	×	Time	=	Interest
Account	Principal		Rate		Time		Interest
Account 1							interest 1
Account 2							interest 2

Understanding Algebra

Reminder:

interest = principal × rate × time

Understanding Algebra

In money problems, *"rate"* applies to rate of interest and is expressed as a decimal.

In context, it is easily distinguished from *rate* used in motion problems.

EXAMPLE 3 Investments Carmine DeSanto has $15,000 to invest. He is considering two investments. One is a loan he can make to another party that pays him 8% simple interest for a year. A second investment is a 1-year certificate of deposit that pays 5%. Carmine decides that he wants to place some money in each investment, and he wants to earn a total of $1125 interest in 1 year from the two investments. How much money should Carmine put in each investment?

Solution Understand and Translate

$$\text{Let } x = \text{amount to be invested at 5\%.}$$
$$\text{Then } 15{,}000 - x = \text{amount to be invested at 8\%.}$$

We use the simple interest formula, interest = principal · rate · time, to solve this problem.

Account	Principal	Rate	Time	Interest
CD	x	0.05	1	$0.05x$
Loan	$15{,}000 - x$	0.08	1	$0.08(15{,}000 - x)$

Since the sum of the interest from the two investments is \$1125, we write the equation

$$\begin{pmatrix}\text{interest from}\\ \text{5\% CD}\end{pmatrix} + \begin{pmatrix}\text{interest from}\\ \text{8\% investment}\end{pmatrix} = \text{total interest}$$

Carry Out

$$\begin{aligned} 0.05x + 0.08(15{,}000 - x) &= 1125\\ 0.05x + 0.08(15{,}000) - 0.08(x) &= 1125\\ 0.05x + 1200 - 0.08x &= 1125\\ -0.03x + 1200 &= 1125\\ -0.03x &= -75\\ x = \frac{-75}{-0.03} &= 2500 \end{aligned}$$

Check and Answer Thus, \$2500 should be invested at 5% interest. The amount to be invested at 8% is

$$15{,}000 - x = 15{,}000 - 2500 = 12{,}500$$

The total amount invested is \$2500 + \$12,500 = \$15,000, which checks with the information given.

Now Try Exercise 33

In Example 3, we let x represent the amount invested at 5%. If we had let x represent the amount invested at 8%, the answer would not have changed. Rework Example 3 now, letting x represent the amount invested at 8%.

In other types of problems involving two amounts of money, we generally set up similar tables, as illustrated in the next example.

EXAMPLE 4 **Rocking Chairs** Johnson's Patio Furniture Store sells two types of rocking chairs. The single-person rocking chair sells for \$130 each and the two-person rocking chair sells for \$240 each. On a given day 10 rocking chairs were sold for a total of \$1740. Determine the number of single-person and the number of two-person rocking chairs that were sold.

Solution Understand and Translate We are asked to find the number of each type of rocking chair sold.

$$\text{Let } x = \text{number of single-person rocking chairs sold.}$$
$$\text{Then } 10 - x = \text{number of two-person rocking chairs sold.}$$

The income received from the sale of the single-person rocking chairs is found by multiplying the number of single-person rocking chairs sold by the cost of a single-person rocking chair. The income received from the sale of the two-person rocking chairs is found by multiplying the number of two-person rocking chairs sold by the cost of a two-person rocking chair.

$$\begin{pmatrix}\text{Number of}\\ \text{Rocking chairs}\end{pmatrix} \times \begin{pmatrix}\text{Cost of}\\ \text{Rocking chairs}\end{pmatrix} = \begin{pmatrix}\text{Income from}\\ \text{Rocking chairs}\end{pmatrix}$$

Rocking Chair	Number of Rocking Chairs	Cost	Income from Rocking Chairs
Single	x	130	$130x$
Double	$10 - x$	240	$240(10 - x)$

$$\begin{pmatrix}\text{income from}\\ \text{single-person}\\ \text{rocking chairs}\end{pmatrix} + \begin{pmatrix}\text{income from}\\ \text{two-person}\\ \text{rocking chairs}\end{pmatrix} = \text{total income}$$

Carry Out

$$\begin{aligned} 130x + 240(10 - x) &= 1740 \\ 130x + 2400 - 240x &= 1740 \\ -110x + 2400 &= 1740 \\ -110x &= -660 \\ x &= \frac{-660}{-110} = 6 \end{aligned}$$

Check and Answer Six single-person rocking chairs and $10 - 6$ or 4 two-person rocking chairs were sold.

Check

$$\begin{aligned} \text{income from 6 single-person rocking chairs} &= 780 \\ \text{income from 4 two-person rocking chairs} &= \underline{960} \\ \text{total} &= 1740 \quad \text{True} \end{aligned}$$

Now Try Exercise 43

3 Solve Mixture Problems