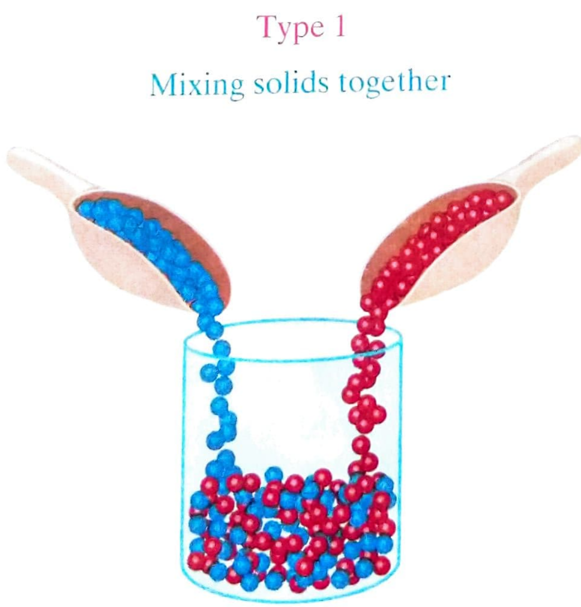
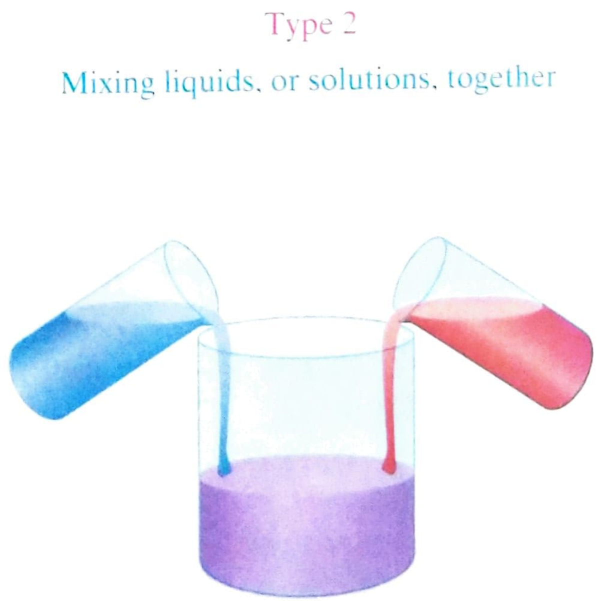
3 Solve Mixture Problems

Any problem in which two or more quantities are combined to produce a single quantity or a single quantity is separated into two or more quantities may be considered a **mixture problem**.

Mixture problems in this section will generally be one of two types. In one type, we will mix two solids, as illustrated in **Figure 3.9a**, and be concerned about the value or cost of the mixture. In the second type, we will mix two liquids or solutions, as illustrated in **Figure 3.9b**, and be concerned about the content or strength of the mixture.



(a) Concerned about the value or the cost of the mixture



(b) Concerned about the content or the strength of the mixture

FIGURE 3.9

As we did with motion problems involving two rates and with money problems, we will use a table to help analyze mixture problems.

When we construct a table for mixture problems, our table will generally have three rows. One row will be for each of the two individual items being mixed, and the third row will be for the mixture of the two items.

Understanding Algebra

If we know the total weight of two items is 10 pounds and one item weighs x pounds, then the second item weighs $(10 - x)$ pounds.

Reason:

item 1 +	item 2	= total
x	$+ (10 - x)$	$= 10$

Type 1—Mixing Solids

When working with mixture problems involving solids, we generally use the fact that the value (or cost) of one part of the mixture plus the value (or cost) of the second part of the mixture is equal to the total value (or total cost) of the mixture.

When we are combining two solid items and are interested in the *value* of the mixture, the following table, or a variation of it, is often used.

Quantity × Price (per unit) = Value of Item

Item	Quantity	Price	Value of Item
Item 1			value of item 1
Item 2			value of item 2
Mixture			value of mixture

When we use this table, we generally use the following formula to solve the problem.

value of item 1 + value of item 2 = value of mixture

Now let's look at a mixture problem where we discuss the value or cost of the mixture.

EXAMPLE 5 **Grass Seed** Scott's Family grass seed sells for \$2.65 per pound, and Scott's Spot Filler grass seed sells for \$2.30 per pound. How many pounds of each should be mixed to get a 10-pound mixture that sells for \$2.40 per pound?

Solution **Understand and Translate** We are asked to find the number of pounds of each type of grass seed.

Let x = number of pounds of Family grass seed.

Then $10 - x$ = number of pounds of Spot Filler grass seed.

We make a sketch of the situation (**Fig. 3.10**), then construct a table.

FIGURE 3.10

The cost or value of the seeds is found by multiplying the number of pounds by the price per pound.

Type of Seed	Number of Pounds	Cost per Pound	Cost of Seed
Family	x	2.65	$2.65x$
Spot Filler	$10 - x$	2.30	$2.30(10 - x)$
Mixture	10	2.40	$2.40(10)$

$$\begin{pmatrix}\text{cost of}\\ \text{Family Seed}\end{pmatrix} + \begin{pmatrix}\text{cost of Spot}\\ \text{Filler Seed}\end{pmatrix} = \text{cost of mixture}$$

$$2.65x + 2.30(10 - x) = 2.40(10)$$

Carry Out

$$2.65x + 23.0 - 2.30x = 24.0$$
$$0.35x + 23.0 = 24.0$$
$$0.35x = 1.00$$
$$x \approx 2.86$$

Answer Thus, about 2.86 pounds of the Family grass seed must be mixed with $10 - x$ or $10 - 2.86 = 7.14$ pounds of the Spot Filler grass seeds to make a mixture that sells for \$2.40 a pound.

Now Try Exercise 47

Type 2—Mixing Solutions

We generally solve mixture problems involving solutions by using the fact that the amount of one part of the mixture plus the amount of the second part of the mixture is equal to the total amount of the mixture.

When working with solutions, we use the following formula: *amount of substance in the solution = quantity of solution × strength of solution (in percent written as a decimal)*. When we are mixing two quantities and are interested in the composition of the mixture, we generally use the following table or a variation of the table.

	Quantity ×	Strength =	Amount of Substance
Solution	Quantity	Strength	Amount of Substance
Solution 1			amount of substance in solution 1
Solution 2			amount of substance in solution 2
Mixture			amount of substance in mixture

When using this table, we generally use the following formula to solve the problem.

$$\left(\begin{array}{c}\text{amount of substance}\\ \text{in solution 1}\end{array}\right) + \left(\begin{array}{c}\text{amount of substance}\\ \text{in solution 2}\end{array}\right) = \left(\begin{array}{c}\text{amount of substance}\\ \text{in mixture}\end{array}\right)$$

Let us now look at an example of a mixture problem where two solutions are combined.

EXAMPLE 6 **Mixing Acid Solutions** Mr. Dave Lumsford needs a 10% acetic acid solution for a chemistry experiment. After checking the store room, he finds that there are only 5% and 20% acetic acid solutions available. Mr. Lumsford decides to make the 10% solution by combining the 5% and 20% solutions. How many liters of the 5% solution must he add to 8 liters of the 20% solution to get a solution that is 10% acetic acid?

Solution **Understand and Translate** We are asked to find the number of liters of the 5% acetic acid solution to mix with 8 liters of the 20% acetic acid solution.

Let x = number of liters of 5% acetic acid solution.

Let's draw a sketch of the problem (**Fig. 3.11**).

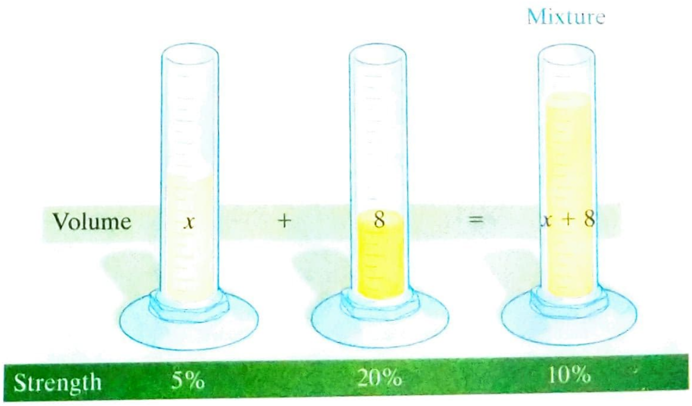


FIGURE 3.11

The amount of acid in a given solution is found by multiplying the number of liters by the percent strength.

Solution	Liters	Strength	Amount of Acetic Acid
5%	x	0.05	$0.05x$
20%	8	0.20	0.20(8)
Mixture	$x + 8$	0.10	$0.10(x + 8)$

$$\begin{pmatrix}\text{amount of acid}\\ \text{in 5\% solution}\end{pmatrix} + \begin{pmatrix}\text{amount of acid}\\ \text{in 20\% solution}\end{pmatrix} = \begin{pmatrix}\text{amount of acid}\\ \text{in 10\% mixture}\end{pmatrix}$$

$$0.05x \quad + \quad 0.20(8) = 0.10(x + 8)$$

Carry Out

$$0.05x + 1.6 = 0.10x + 0.8$$
$$0.05x + 0.8 = 0.10x$$
$$0.8 = 0.05x$$
$$\frac{0.8}{0.05} = x$$
$$16 = x$$

Answer Sixteen liters of 5% acetic acid solution must be added to the 8 liters of 20% acetic acid solution to get a 10% acetic acid solution. The total number of liters that will be obtained is 16 + 8 or 24.

Now Try Exercise 55

EXAMPLE 7 Nicole Pappas, a medical researcher, has 40% and 5% solutions of phenobarbital. How much of each solution must she mix to get 0.6 liter of a 20% phenobarbital solution?

Solution **Understand and Translate** We are asked to find how much of the 40% and 5% phenobarbital solutions must be mixed to get 0.6 liter of a 20% solution. We can choose to let x be the amount of either the 40% or the 5% solution. We will choose as follows:

Let x = number of liters of the 40% solution.

Then $0.6 - x$ = number of liters of the 5% solution.

Remember from Section 3.1 that if a total of 0.6 liter is divided in two, if one part is x, the other part is $0.6 - x$.

Let's draw a sketch of the problem (**Fig. 3.12**).

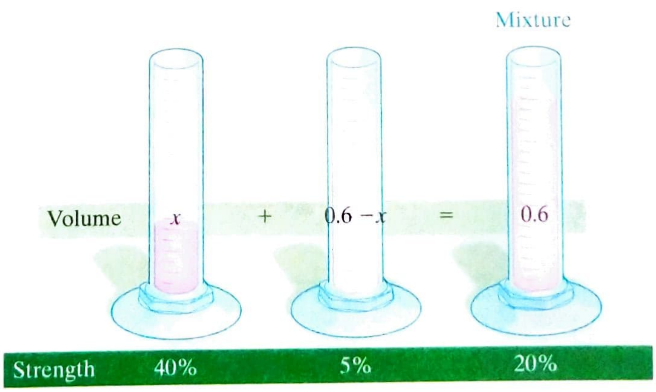


FIGURE 3.12

The amount of phenobarbital in a given solution is found by multiplying the number of liters by the percent strength.

Solution	Liters	Strength	Amount of Phenobarbital
40%	x	0.40	$0.40x$
5%	$0.6 - x$	0.05	$0.05(0.6 - x)$
Mixture	0.6	0.20	$0.6(0.20)$

$$\begin{pmatrix}\text{amount of phenobarbital}\\ \text{in 40\% solution}\end{pmatrix} + \begin{pmatrix}\text{amount of phenobarbital}\\ \text{in 5\% solution}\end{pmatrix} = \begin{pmatrix}\text{amount of phenobarbital}\\ \text{in mixture}\end{pmatrix}$$

$$0.40x \quad + \quad 0.05(0.6 - x) = (0.6)(0.20)$$

Carry Out

$$0.40x + 0.03 - 0.05x = 0.12$$
$$0.35x + 0.03 = 0.12$$
$$0.35x = 0.09$$
$$x \approx 0.26$$

Answer Since the answer was less than 0.6 liter, the answer is reasonable. About 0.26 liter of the 40% solution must be mixed with about $0.6 - x = 0.60 - 0.26 = 0.34$ liter of the 5% solution to get 0.6 liter of the 20% mixture.

Now Try Exercise 63

In Example 7, we chose to let x = number of liters of the 40% solution. We could have selected to let x = number of liters of the 5% solution. Then $0.6 - x$ would be the number of liters of the 40% solution. Had you worked the problem out like this, you would have found that x was approximately 0.34 liter. Try reworking Example 7 now letting x = number of liters of the 5% solution.

EXAMPLE 8 An orange punch contains 4% orange juice. If 5 ounces of water is added to 8 ounces of the punch, determine the percent of orange juice in the mixture.

Solution **Understand and Translate** We are asked to find the percent of orange juice in the mixture.

Let x = percent of orange juice in the mixture.

We will again set up a table.

Solution	Ounces	Percent of Juice	Amount of Juice
Punch	8	0.04	8(0.04)
Water	5	0.00	5(0.00)
Mixture	13	x	$13x$

$$\begin{pmatrix}\text{amount of juice}\\ \text{in punch}\end{pmatrix} + \begin{pmatrix}\text{amount of juice}\\ \text{in water}\end{pmatrix} = \begin{pmatrix}\text{amount of juice}\\ \text{in mixture}\end{pmatrix}$$
$$8(0.04) + 5(0.00) = 13x$$

Carry Out

$$0.32 + 0.00 = 13x$$
$$0.32 = 13x$$
$$0.025 \approx x$$

Answer Therefore, the percent of juice in the mixture is about 2.5%.

Now Try Exercise 59

EXERCISE SET 3.4

MathXL® MyMathLab

Warm-Up Exercises

Fill in the blanks with the appropriate word, phrase, or symbol(s) from the following list.

$d = r \cdot t$	adding	subtracting	multiplying
$i = p \cdot r \cdot t$	$8 - x$	the percent	

1. Solving a motion problem when the two items are traveling in the same direction usually involves ______________ the distances.
2. In a mixture problem, if there is a total of 8 liters and the amount of one unknown is x, then the amount of the other unknown is ______________.
3. A formula important in the solution of motion problems is ______________.
4. Solving a money problem may include using the formula ______________.
5. Solving a motion problem when the two items are traveling in different directions usually involves ______________ the distances.
6. To find the amount of alcohol in an 8-liter solution, we multiply the quantity of solution times ______________ of alcohol in the solution.