8. $\dfrac{x^3 + 27}{x^2 - 2x - 15} \cdot \dfrac{x^2 - 7x + 10}{x^2 - 3x + 9}$

9. $\dfrac{5x - 1}{x^2 + 11x + 10} \div \dfrac{10x - 2}{x^2 + 17x + 70}$

10. $\dfrac{5x^2 + 7x + 2}{x^2 + 6x + 5} \div \dfrac{7x^2 - 39x - 18}{x^2 - x - 30}$

Add or subtract as indicated.

11. $\dfrac{x^2}{x + 6} - \dfrac{36}{x + 6}$

12. $\dfrac{2x^2 - 2x}{2x + 5} + \dfrac{x - 15}{2x + 5}$

13. $\dfrac{3t^2 - t}{4t^2 - 9t + 2} - \dfrac{3t + 4}{4t^2 - 9t + 2}$

Find the least common denominator.

14. $\dfrac{2m}{6m^2 + 3m} + \dfrac{m + 7}{2m + 1}$

15. $\dfrac{9x + 8}{2x^2 - 5x - 12} + \dfrac{2x + 3}{x^2 - 9x + 20}$

For Exercises 16–19, add or subtract as indicated.

16. $\dfrac{x + 1}{2x} + \dfrac{4x - 3}{5x}$

17. $\dfrac{2a + 5}{a + 3} - \dfrac{3a + 1}{a - 4}$

18. $\dfrac{x^2 + 5}{2x^2 + 13x + 6} + \dfrac{3x - 1}{2x + 1}$

19. $\dfrac{x}{x^2 + 3x + 2} - \dfrac{4}{x^2 - x - 6}$

20. To add the rational expressions $\dfrac{7}{x + 1} + \dfrac{8}{x}$, Samuel Ditsi decided to add both numerators and then add both denominators to get $\dfrac{7 + 8}{(x + 1) + x}$, which simplified to $\dfrac{15}{2x + 1}$. This procedure is wrong. Why is it wrong? Explain your answer. Then add the rational expressions $\dfrac{7}{x + 1} + \dfrac{8}{x}$ correctly.

6.5 Complex Fractions

1 Simplify complex fractions by combining terms.

2 Simplify complex fractions using multiplication first to clear fractions.

1 Simplify Complex Fractions by Combining Terms

Complex Fraction

A **complex fraction** is one that has a fraction in its numerator or its denominator or in both its numerator and denominator.

Examples of Complex Fractions

$$\frac{\frac{3}{5}}{7} \qquad \frac{\frac{x + 9}{x}}{4x} \qquad \frac{x}{\frac{y}{x + 1}} \qquad \frac{\frac{a + b}{a}}{\frac{a - b}{b}}$$

The expression above the **main fraction bar** is the numerator of the complex fraction, and the expression below the main fraction bar is the denominator of the complex fraction.

$$\frac{\frac{a + b}{a}}{\frac{a - b}{b}}$$

Numerator of complex fraction: $\frac{a + b}{a}$

← Main fraction bar

Denominator of complex fraction: $\frac{a - b}{b}$

There are two methods to simplify complex fractions. The first reinforces many of the concepts used in this chapter because we may need to add, subtract, multiply, and divide simpler fractions as we simplify the complex fraction. Many students prefer to use the second method because the answer may be obtained more quickly.

Method 1—To Simplify a Complex Fraction by Combining Terms

1. Add or subtract the fractions in both the numerator and denominator of the complex fraction to obtain single fractions in both the numerator and the denominator.
2. Multiply the fraction in the numerator by the reciprocal of the fraction in the denominator.
3. Simplify further if possible.

Understanding Algebra

The complex fraction

$$\frac{\frac{a + b}{c}}{\frac{d + e}{f}}$$

is another way to represent the division

$$\frac{a + b}{c} \div \frac{d + e}{f}.$$

We perform the division by multiplying the first fraction by the reciprocal of the second fraction:

$$\frac{a + b}{c} \cdot \frac{f}{d + e}$$

EXAMPLE 1 Simplify $\dfrac{\dfrac{ab^2}{c^3}}{\dfrac{a}{bc^2}}$.

Solution Since both numerator and denominator are already single fractions, we begin with step 2.

$$\frac{\frac{ab^2}{c^3}}{\frac{a}{bc^2}} = \frac{ab^2}{c^3} \div \frac{a}{bc^2} \qquad \text{A fraction bar means "divided by."}$$

$$= \frac{ab^2}{c^3} \cdot \frac{bc^2}{a} \qquad \text{Multiply by } \frac{bc^2}{a}\text{, the reciprocal of } \frac{a}{bc^2}.$$

$$= \frac{b^3}{c} \qquad \text{Both } a \text{ and } c^2 \text{ were factored out of the numerator and denominator.}$$

Thus the expression simplifies to $\dfrac{b^3}{c}$.

Now Try Exercise 11

EXAMPLE 2 Simplify $\dfrac{a + \dfrac{1}{x}}{x + \dfrac{1}{a}}$.

Solution Step 1 is to express the numerator and denominator as single fractions. To obtain one fraction in the numerator, we notice that the LCD is x. Multiply a by $\dfrac{x}{x}$.

$$a + \frac{1}{x} = a \cdot \frac{x}{x} + \frac{1}{x} = \frac{ax+1}{x} \qquad \text{We use this as the numerator.}$$

To obtain one fraction in the denominator, we notice that the LCD is a. Multiply x by $\dfrac{a}{a}$.

$$x + \frac{1}{a} = x \cdot \frac{a}{a} + \frac{1}{a} = \frac{ax+1}{a} \qquad \text{We use this as the denominator.}$$

So
$$\frac{a + \frac{1}{x}}{x + \frac{1}{a}} = \frac{\frac{ax+1}{x}}{\frac{ax+1}{a}} = \frac{ax+1}{x} \div \frac{ax+1}{a}$$

$$= \frac{ax+1}{x} \cdot \frac{a}{ax+1} = \frac{a}{x}$$

Now Try Exercise 25

In Example 4 we will rework Example 2 using the second method.

2 Simplify Complex Fractions Using Multiplication First to Clear Fractions

Method 2—To Simplify a Complex Fraction Using Multiplication First

1. Find the least common denominator of *all* the denominators appearing in the complex fraction.
2. Multiply both the numerator and denominator of the complex fraction by the LCD found in step 1.
3. Simplify when possible.

EXAMPLE 3 Simplify $\dfrac{\frac{2}{3}+\frac{1}{5}}{\frac{4}{5}-\frac{1}{3}}$.

Solution The denominators in the complex fraction are 3 and 5. Multiply both the numerator and denominator of the complex fraction by 15, the LCD of the complex fraction.

$$\frac{\frac{2}{3}+\frac{1}{5}}{\frac{4}{5}-\frac{1}{3}} = \frac{15}{15}\cdot\frac{\left(\frac{2}{3}+\frac{1}{5}\right)}{\left(\frac{4}{5}-\frac{1}{3}\right)} = \frac{15\left(\frac{2}{3}\right)+15\left(\frac{1}{5}\right)}{15\left(\frac{4}{5}\right)-15\left(\frac{1}{3}\right)}$$

Now simplify.

$$= \frac{10+3}{12-5} = \frac{13}{7}$$

Now Try Exercise 9

Understanding Algebra

The LCD of the complex fraction

$$\frac{\frac{a+b}{c}}{\frac{d+e}{f}}$$

is cf (c from the numerator and f from the denominator). To eliminate fractions within the complex fraction, we multiply both the numerator and the denominator by cf:

$$\frac{cf}{cf}\cdot\frac{\left(\frac{a+b}{c}\right)}{\left(\frac{d+e}{f}\right)} = \frac{(a+b)f}{(d+e)c}$$

Now we will rework Example 2 using method 2.

EXAMPLE 4 Simplify $\dfrac{a+\frac{1}{x}}{x+\frac{1}{a}}$.

Solution The denominators in the complex fraction are x and a. Multiply both the numerator and denominator of the complex fraction by ax, the LCD of the complex fraction.

$$\frac{a+\frac{1}{x}}{x+\frac{1}{a}} = \frac{ax}{ax}\cdot\frac{\left(a+\frac{1}{x}\right)}{\left(x+\frac{1}{a}\right)} = \frac{a^2x+a}{ax^2+x}$$

$$= \frac{a\,\cancel{(ax+1)}}{x\,\cancel{(ax+1)}} = \frac{a}{x}$$

Now Try Exercise 25

Note that the answers to Examples 2 and 4 are the same.

EXAMPLE 5 Simplify $\dfrac{y^2}{\frac{1}{x}+\frac{1}{y}}$.

Solution The denominators in the complex fraction are x and y. Multiply both the numerator and denominator of the complex fraction by xy, the LCD of the complex fraction.

$$\frac{y^2}{\frac{1}{x}+\frac{1}{y}} = \frac{xy}{xy}\cdot\frac{y^2}{\left(\frac{1}{x}+\frac{1}{y}\right)}$$

$$= \frac{xy^3}{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)}$$

$$= \frac{xy^3}{y+x}$$

Now Try Exercise 33

Helpful Hint

We have presented two methods for simplifying complex fractions. Which method should you use? Although either method can be used to simplify complex fractions, most students prefer to use method 1 when both the numerator and denominator consist of a single term, as in Example 1. When the complex fraction has a sum or difference of expressions in either the numerator or denominator, as in Examples 2, 3, 4, or 5, most students prefer to use method 2.

EXERCISE SET 6.5

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Warm-Up Exercises

Fill in the blanks with the appropriate word, phrase, or symbol(s) from the following list.

complex fraction denominator numerator 5 $\frac{5}{3}$ $x^2 + 5x + 6$

1. The numerator of the complex fraction $\dfrac{\frac{5}{3}}{x^2 + 5x + 6}$ is __________.

2. A fraction that contains a fraction in its numerator or its denominator or in both is called a __________.

3. The numerator of the complex fraction $\dfrac{5}{\dfrac{3}{x^2 + 5x + 6}}$ is __________.

4. To simplify a complex fraction using multiplication, multiply both the numerator and __________ of the complex fraction by the LCD of the complex fraction.

Practice the Skills

Simplify.

5. $\dfrac{1 + \frac{2}{3}}{5 + \frac{1}{3}}$

6. $\dfrac{2 + \frac{4}{5}}{1 - \frac{9}{16}}$

7. $\dfrac{2 + \frac{3}{8}}{1 + \frac{1}{3}}$

8. $\dfrac{\frac{1}{4} + \frac{5}{6}}{\frac{2}{3} + \frac{3}{5}}$

9. $\dfrac{\frac{2}{3} + \frac{1}{4}}{\frac{5}{6} - \frac{1}{3}}$

10. $\dfrac{\frac{5}{8} + \frac{1}{3}}{\frac{11}{12} - \frac{1}{6}}$

11. $\dfrac{\frac{xy^2}{7}}{\frac{3}{x^2}}$

12. $\dfrac{\frac{11a}{b^3}}{\frac{b^2}{4}}$

13. $\dfrac{\frac{6a^2b}{7}}{\frac{9ac^2}{b^2}}$

14. $\dfrac{\frac{18x^4}{5y^4z^5}}{\frac{9xy^2}{15z^5}}$

15. $\dfrac{a - \frac{a}{b}}{\frac{3 + a}{b}}$

16. $\dfrac{a + \frac{2}{b}}{\frac{a}{b}}$

17. $\dfrac{\frac{9}{t} + \frac{3}{t^2}}{3 + \frac{1}{t}}$

18. $\dfrac{\frac{4}{a} + \frac{1}{2a}}{a + \frac{a}{2}}$

19. $\dfrac{5 - \frac{1}{x}}{4 - \frac{1}{x}}$

20. $\dfrac{\frac{2x}{x - y}}{\frac{x^2}{y}}$

21. $\dfrac{\frac{m}{n} - \frac{n}{m}}{\frac{m + n}{n}}$

22. $\dfrac{5}{\frac{1}{x} + y}$

23. $\dfrac{\frac{a^2}{b} - b}{\frac{b^2}{a} - a}$

24. $\dfrac{\frac{1}{x^2} - \frac{8}{x}}{3 + \frac{1}{x^2}}$

25. $\dfrac{2 - \frac{a}{b}}{\frac{a}{b} - 2}$

26. $\dfrac{\frac{x}{y} - 9}{\frac{-x}{y} + 9}$

27. $\dfrac{\frac{4}{t^2} + \frac{4}{t}}{\frac{4}{t} + \frac{4}{t^2}}$

28. $\dfrac{\frac{a^2 - b^2}{a}}{\frac{a + b}{a^4}}$

29. $\dfrac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{ab}}$

30. $\dfrac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}}$

31. $\dfrac{\frac{a}{b} + \frac{1}{a}}{\frac{b}{a} + \frac{1}{a}}$

32. $\dfrac{\frac{2}{a} + \frac{3}{b}}{\frac{1}{a}}$

33. $\dfrac{x}{\frac{1}{x}-\frac{1}{y}}$

34. $\dfrac{\frac{1}{a}+\frac{1}{b}}{ab}$

35. $\dfrac{\frac{5}{a}+\frac{5}{a^2}}{\frac{5}{b}+\frac{5}{b^2}}$

36. $\dfrac{\frac{x}{y}-\frac{2}{x}}{\frac{y}{x}+\frac{1}{y}}$

Problem Solving

For the complex fractions in Exercises 37–40,

a) *Determine which of the two methods discussed in this section you would use to simplify the fraction. Explain why.*

b) *Simplify by the method you selected in part* **a)**.

c) *Simplify by the method you did not select in part* **a)**. *If your answers to parts* **b)** *and* **c)** *are not the same, explain why.*

37. $\dfrac{5+\frac{3}{5}}{\frac{1}{8}-4}$

38. $\dfrac{\frac{x+y}{x^3}-\frac{1}{x}}{\frac{x-y}{x^5}+5}$

39. $\dfrac{\frac{x-y}{x+y}+\frac{6}{x+y}}{2-\frac{7}{x+y}}$

40. $\dfrac{\frac{25}{x-y}+\frac{2}{x+y}}{\frac{5}{x-y}-\frac{3}{x+y}}$

In Exercises 41 and 42, **a)** *write the complex fraction, and* **b)** *simplify the complex fraction.*

41. The numerator of the complex fraction consists of one term: 5 is divided by $12x$. The denominator of the complex fraction consists of two terms: 4 divided by $3x$ is subtracted from 8 divided by x^2.

42. The numerator of the complex fraction consists of two terms: 3 divided by $2x$ is subtracted from 6 divided by x. The denominator of the complex fraction consists of two terms: the sum of x and the quantity 1 divided by x.

Concept/Writing Exercises

43. Explain what a complex number is.

44. **a)** Select the method you prefer to simplify complex fractions and then write down a step-by-step procedure for simplifying complex fractions using that method.

b) Using your answer to part **a)**, simplify $\dfrac{\frac{4}{x}-\frac{3}{y}}{x+\frac{1}{y}}$.

Challenge Problems

Simplify. (Hint: Refer to Section 4.2, which discusses negative exponents.)

45. $\dfrac{x^{-1}+y^{-1}}{3}$

46. $\dfrac{x^{-1}+y^{-1}}{y^{-1}}$

47. $\dfrac{x^{-1}+y^{-1}}{x^{-1}y^{-1}}$

48. $\dfrac{x^{-2}-y^{-2}}{y^{-1}-x^{-1}}$

49. **Jack** The efficiency of a jack, E, is expressed by the formula $E=\dfrac{\frac{1}{2}h}{h+\frac{1}{2}}$, where h is determined by the pitch of the jack's thread. Determine the efficiency of a jack if h is

a) $\frac{2}{3}$ **b)** $\frac{4}{5}$

Simplify.

50. $\dfrac{\frac{x}{y}+\frac{y}{x}+\frac{2}{x}}{\frac{x}{y}+y}$

51. $\dfrac{\frac{a}{b}+b-\frac{1}{a}}{\frac{a}{b^2}-\frac{b}{a}+\frac{3}{a^2}}$

52. $\dfrac{x}{4+\frac{x}{1+x}}$

Cumulative Review Exercises

[2.5] 53. Solve the equation

$$2x-8(5-x)=9x-3(x+2).$$

[4.4] 54. What is a polynomial?

[5.3] 55. Factor $x^2-13x+40$.

[6.4] 56. Subtract $\dfrac{x}{3x^2+17x-6}-\dfrac{2}{x^2+3x-18}$.